L’itinéraire d’un Mathématicien

par

Andrée C. Ehresmann

Diaporama conçu et réalisé par J.P.Vanbremeersch

1. **Youth**

a) 1905-1924 Childhood

Birth in Strasbourg on April 19, 1905 in a modest protestant family;

father gardener, an uncle missionary in China, 1 older brother.

First language: Alsacian dialect, German;

French only after the end of the 1st war

Influenced by anthroposophy (followers of Goethe)

1. **Youth**

b) 1924-29 Formation

Admission to the Ecole Normale Supérieure in 1924.

In the same "promotion": Marcel Brelot and Jean Dieudonne, Raymond Aron (with whom he plays tennis) and Jean-Paul Sartre.

In nearby promotions Henri Cartan and Cavailles (promotion 1923), and Lautman (promotion 1926).

Agrégation in 1927, followed by 1 year Military service, and 1 year teaching in Marocco in a Lycée.

Begins his research in Geometry in 1930, under the direction of Elie Cartan.

the Ecole Normale Supérieure in 1924.

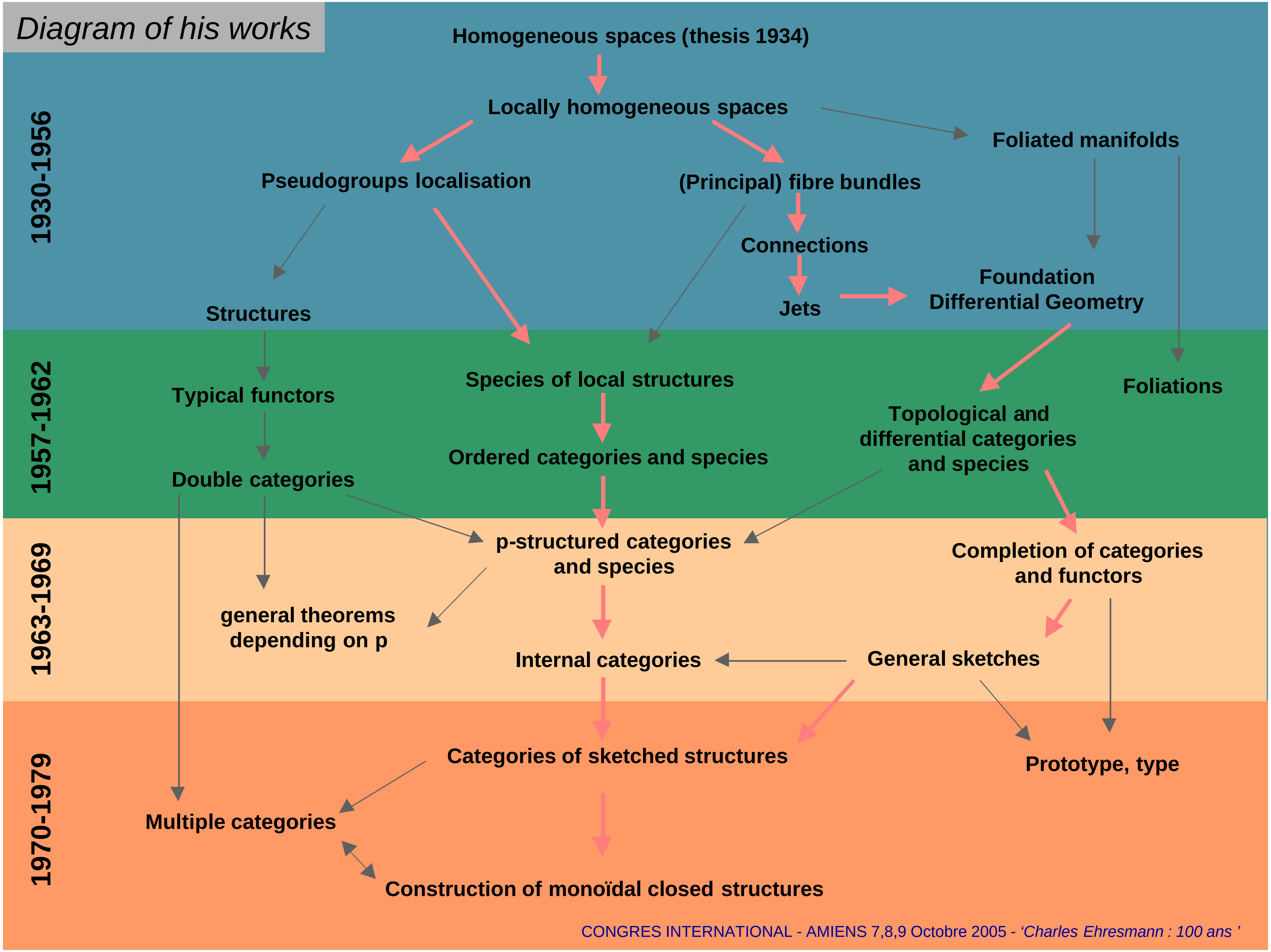


Diagram of his works
1930-1956
1957-1962
1963-1969
1970-1979
Homogeneous spaces (thesis 1934)
Locally homogeneous spaces
Foliated manifolds
Pseudogroups localisation
(Principal) fibre bundles
Connections
Jets
Foundation
Differential Geometry
Structures
Typical functors
Species of local structures
Foliations
Topological and
differential categories
and species
Ordered categories and species
Double categories
p-structured categories
and species
Completion of categories
and functors
general theorems
depending on p
Internal categories
General sketches
Categories of sketched structures
Prototype, type
Multiple categories
Construction of monoïdal closed structures
CONGRES INTERNATIONAL - AMIENS 7,8,9 Octobre 2005 - 'Charles Ehresmann : 100 ans'

2. Before the war: homogeneous and locally homogeneous spaces

a) 1930-34 His thesis: Homogeneous spaces

Research student in Paris

Spends some months in Göttingen to study with Herman Weyl (but does not make a close contact with him), then 2 years (1932-34) in Princeton, where he briefly met Einstein.

1930-1934

Homogeneous spaces (thesis 1934)

His thesis, prepared during his Princeton stay, is the study of the topology and homology of some homogeneous spaces, in particular of Grassman manifolds for which he introduces a new subdivision in cells, later used in the theory of CW-complexes.

A *homogeneous space* is a space with a transitive group of transformations. It defines a geometry in the sense of Klein's program.

Examples: Classical Euclidean geometry as the study of the group formed by the (Euclidean) motions in the plane or in R^3, and of its invariants. Affine geometry, projective geometry

2. Before the war: homogeneous and locally homogeneous spaces

b)1934-39 Locally homogeneous spaces

After obtaining his doctorate in 1934, he remains in Paris with a research allowance from the (ancestor of the) CNRS.

First marriage in 1934.

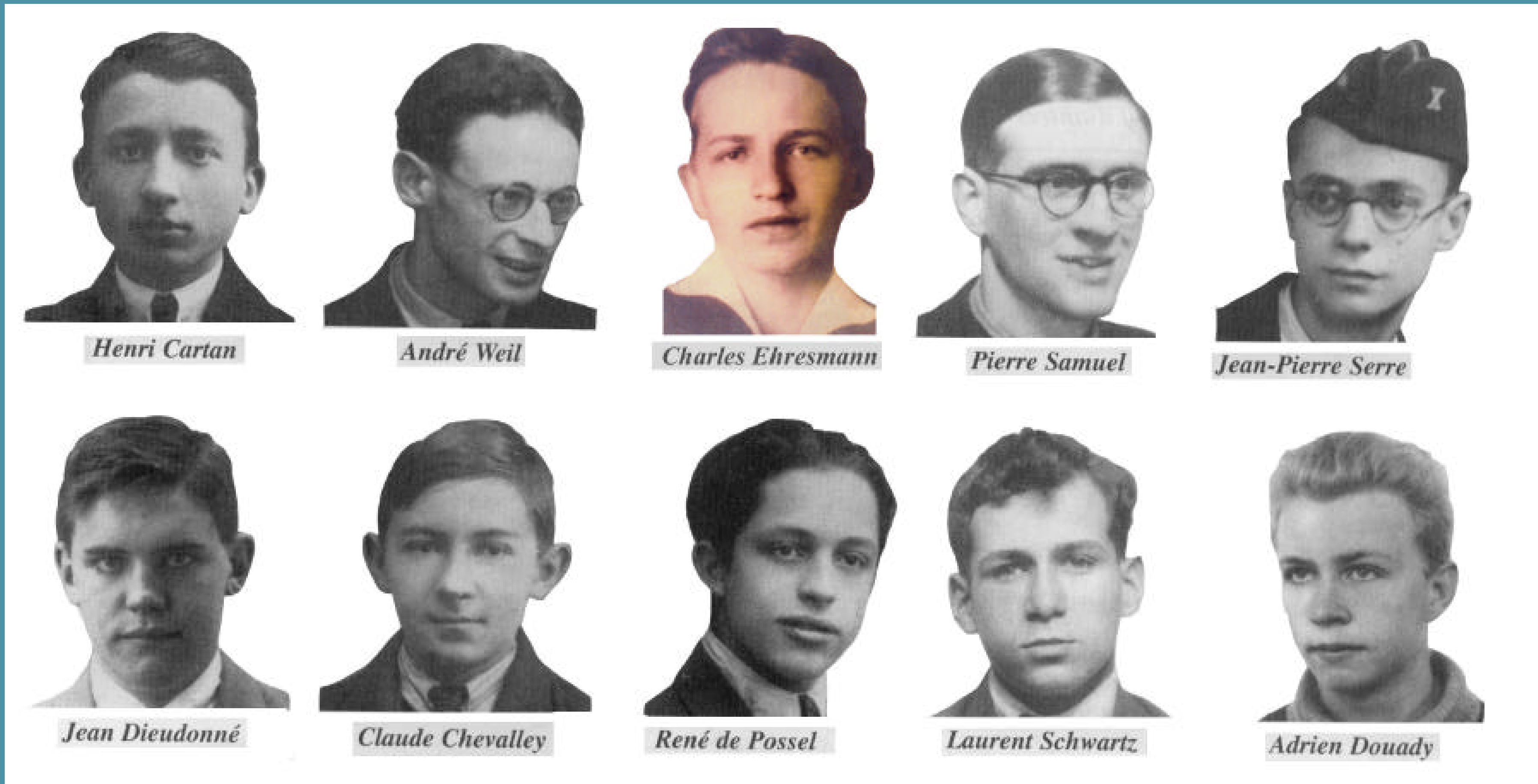
Is a member of Bourbaki and participates actively.

Regular contacts with philosophers, in particular Cavailles and Lautman (cf. Ageron's talk)

Nomination as a Maître de Conférences in Strasbourg in 1938.

De gauche à droite, Simone Weil, Ch. Pisot, A. Weil, J. Dieudonné (assis), C. Chabauty, Ch. Ehresmann, J. Delsarte, au congrès de Dieulefit (Drôme), en 1938.

Some members of Bourbaki



1934-1939

Homogeneous spaces (thesis 1934)

Locally homogeneous spaces

Pseudogroups localisation

Structures

His work on homogeneous spaces is extended to *locally homogeneous spaces*, that is spaces in which each point has an open neighbourhood isomorphic to a given homogeneous space.
Raising the problem of "localisation", he introduces the notion of a *pseudogroup of transformations* (refining a notion of Veblen-Whitehead) and constructs its associated *local structures*, by gluing the charts of an atlas in which the changes of chart belong to the pseudogroup

Applications in Physics:
Locally homogeneous spaces have recently been used in quantum gravity

Pseudogroup and its associated local structure

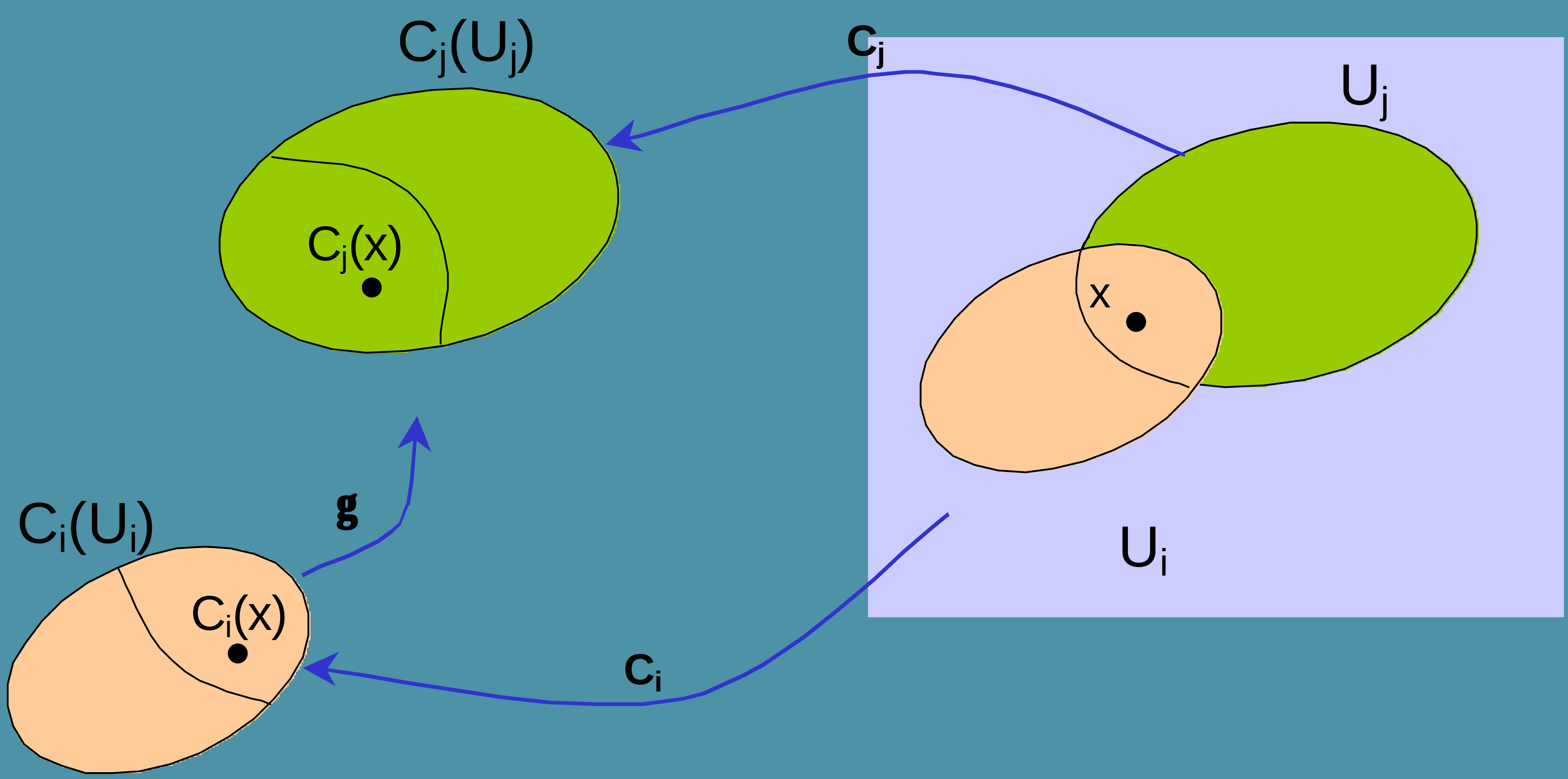


FIGURE 1

E is locally isomorphic to D with respect to a pseudogroup of transformations Γ on D if there is an atlas A = (c_i) such that E is the union of the domains U_i of its charts c_i and all the changes of charts $\gamma = c_j c_i^{-1}$ are in Γ, meaning that $c_j(x) = \gamma(c_i(x))$ for each x in the intersection of U_i and U_j.

The sphere

Examples: the sphere S^2 is locally isomorphic to R^2; more generally, topological or differentiable manifolds are locally isomorphic to an R^n.

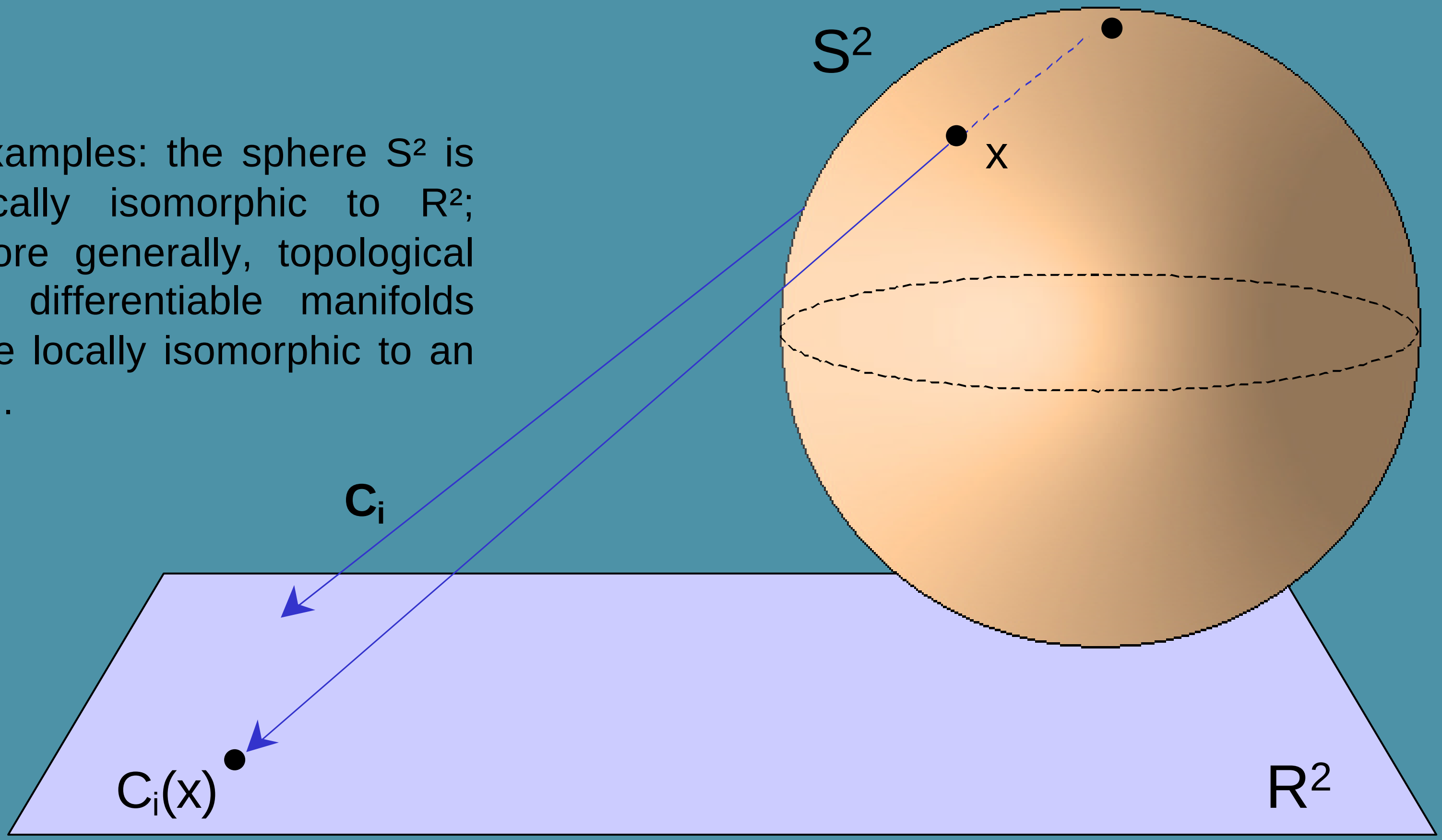


FIGURE 2
The sphere S^2 is defined by an atlas in which one chart is the stereographic projection from the North pole of the sphere except its south pole, another is similarly obtained by inverting the 2 poles..

Applications in Physics: Locally homogeneous spaces have recently been used in quantum gravity

3. From 1939 to 1956

a) During the war: fibre bundles

Up to the armistice, he serves as an officer; after that, he lives in Clermont-Ferrand where the University of Strasbourg has been withdrawn.

1942: birth of Jean-Marc, his unique child.

He actively participates to Bourbaki.

After a raid against the University, he hides

in a small village near Clermont-Ferrand.

1939-1956

Homogeneous spaces (thesis 1934)

↓

Locally homogeneous spaces

↙ ↘

Pseudogroups localisation | **(Principal) fibre bundles**

↓

Structures

Using his theory of localisation, he defines a locally trivial *fibre bundle* with base B and fibre F as a space E locally isomorphic to the product BxF. The introduction of a topological group G acting on F (the "structural group") leads him to the essential notions of a principal bundle (when the fibre is G itself acting on itself) and of its associated G-bundles

A fibre bundle

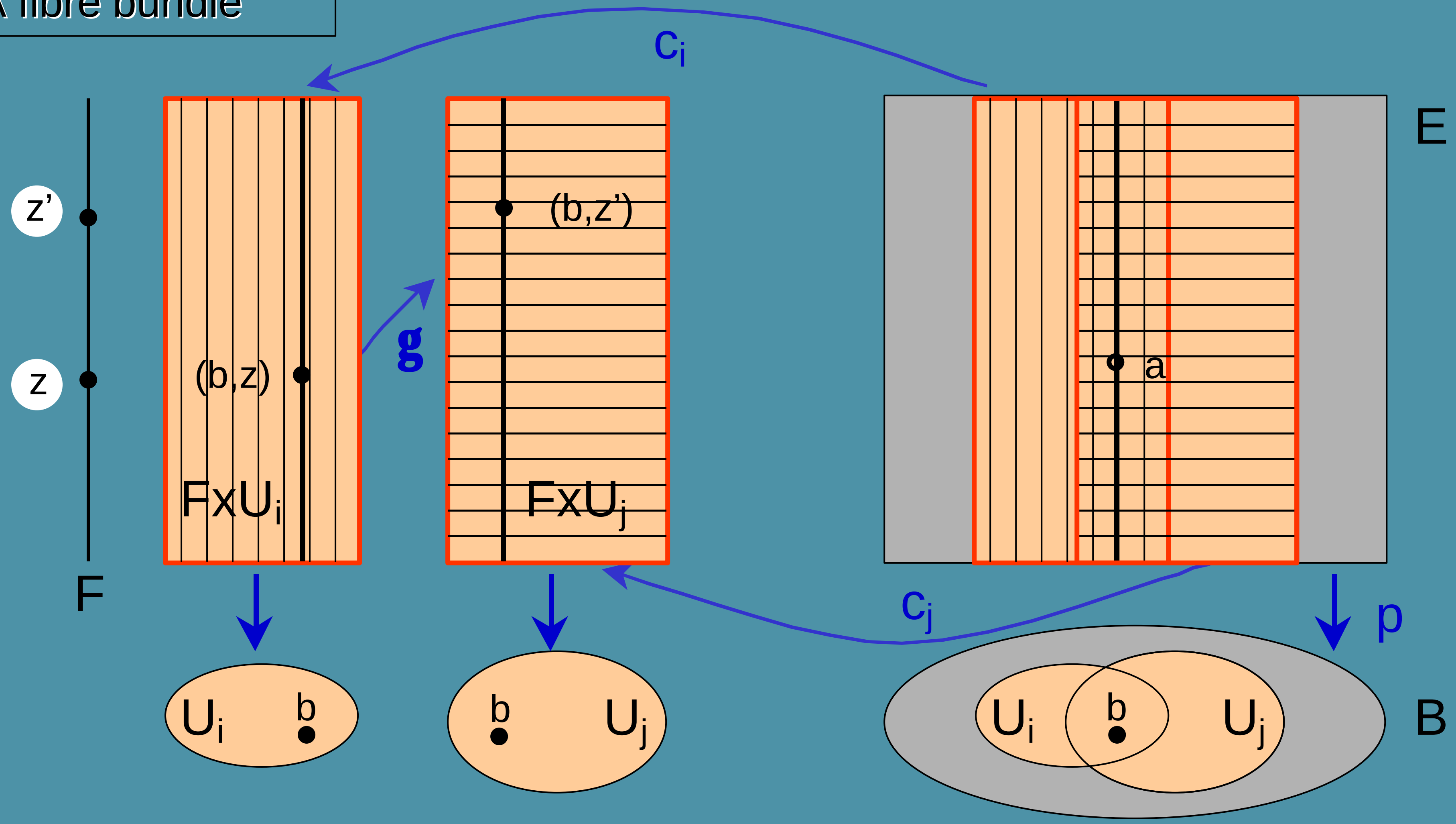
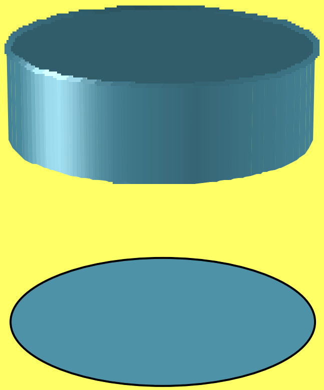


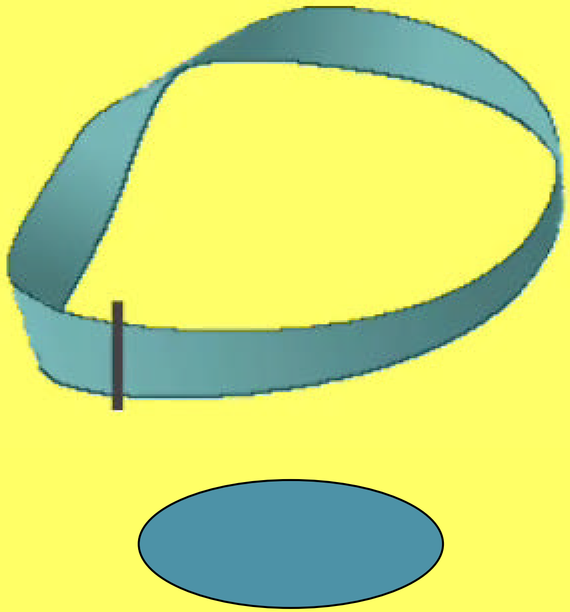
FIGURE 3
E = fibre bundle with base B and fibre F. It is associated to a pseudogroup of transformations on BxF whose transformations **g** are maps from BxU to BxU of the form (b,z) -> (b, z'). It is a G-bundle if G is a group acting on F and if the changes of charts γ are of the form (b,z) -> (b,gz) for some g in G..

Examples of fibre bundles

Examples: A cylinder is a trivial bundle SxI. A Möbius band is a non-trivial bundle, obtained as a "twisted product" of SxI. A magnetic monopole is a fibre bundle on S^3 with base S^1 and fibre S^2.



Cylinder



Möbius

FIGURE 4

A cylinder is a trivial bundle with base the circle S and fibre a segment I. The Möbius band is a non-trivial bundle with the same base and fibre, but obtained by "twisting" the product SxI via the action of the structural group Z_2.

1939-1956

Homogeneous spaces (thesis 1934)

↓

Locally homogeneous spaces

↙ ↘

Pseudogroups localisation | **(Principal) fibre bundles**

↓

Structures

Charles develops the theory of fibre bundles in several papers. In particular he proves various lifting of homotopies theorems. He raises the problem of reduction of the structural group and solves it in some cases, for instance by defining quasi-complex manifolds.

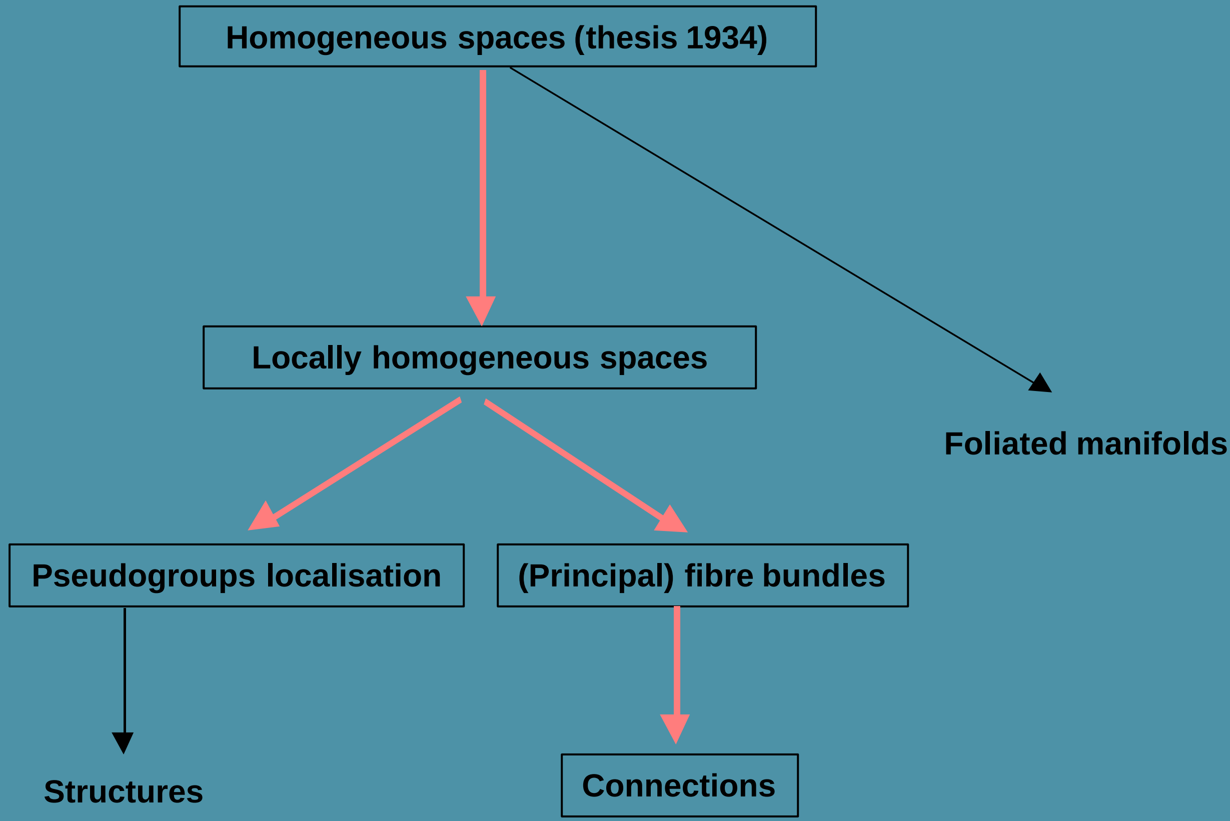
3. From 1939 to 1956

b) From 1946 to 1950: Connections and Foliations

Professor at the University of Strasbourg, he organizes a Seminar and several international meetings in this town.

He directs the theses of Reeb, Wu Wen Tsun and P. Libermann.

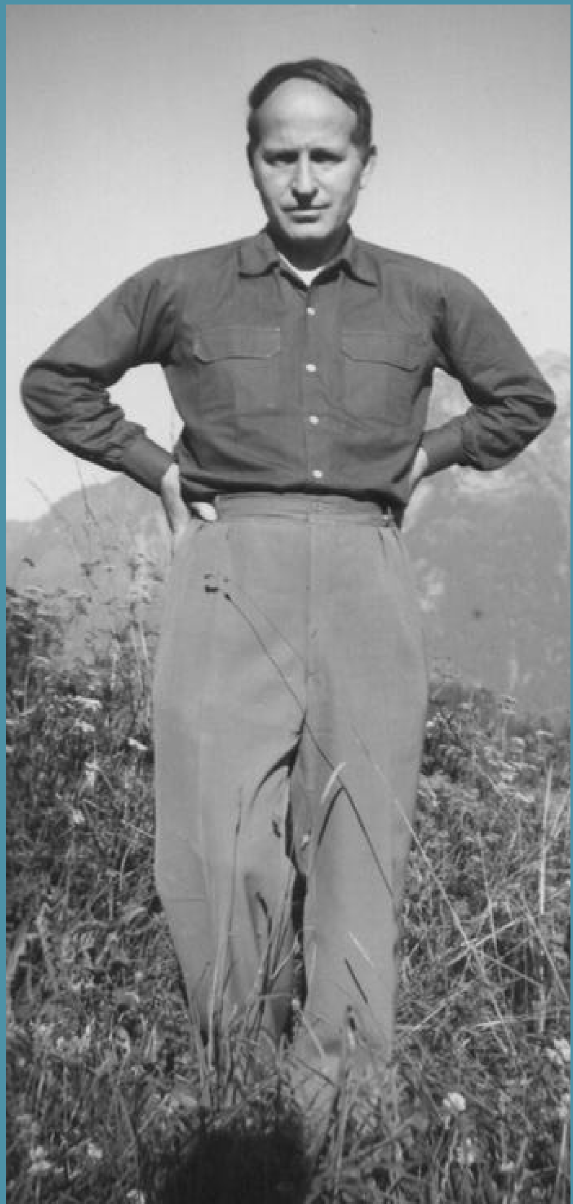
1939-1956

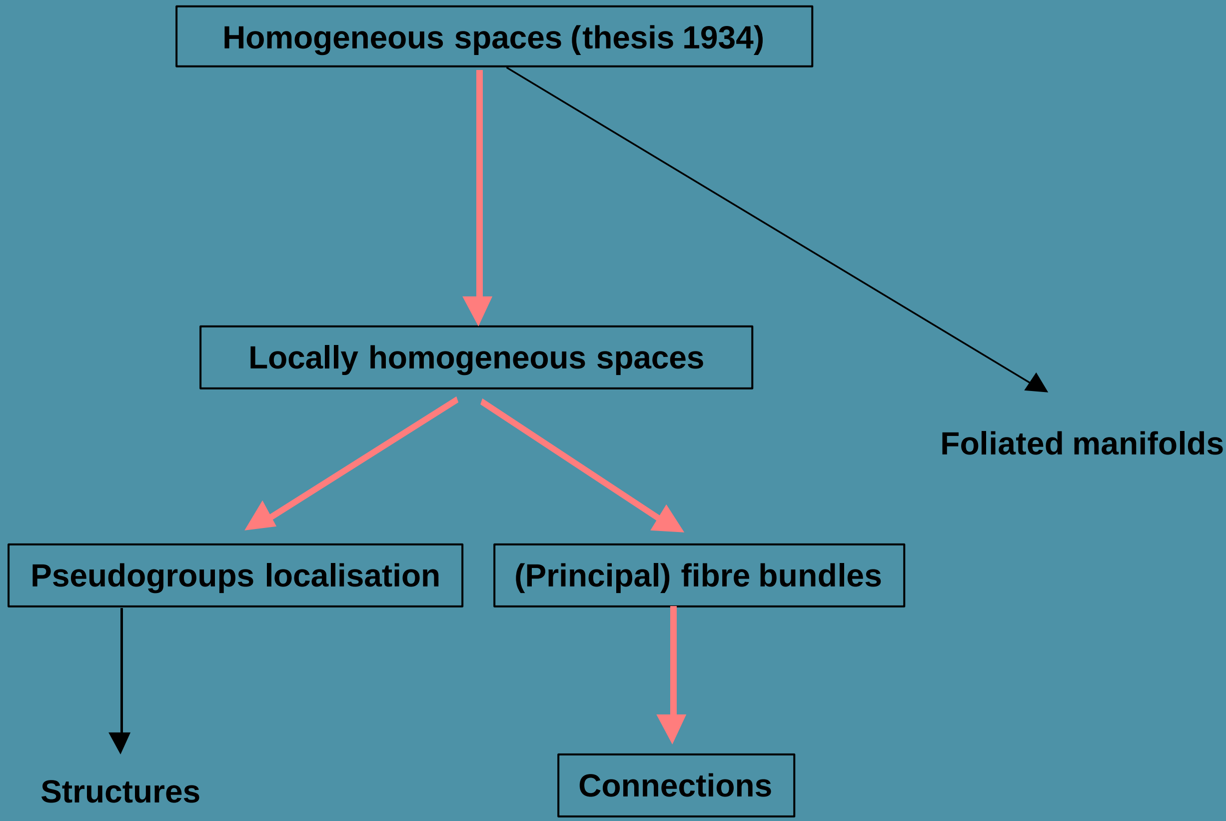


Pursuing his theory of fibre bundles, he develops the now classical theory of a connection and its curvature. Roughly a connection on a principal bundle is a field of contact elements transverse to the fibres.
Example: a helix on a cylinder is an example of a non-integrable connection.

In this period he also introduces the notion of a *foliated manifold*, developed in Reeb's thesis. Later on he will define more general *foliations* (1956) and study their holonomy in a long paper (1961).

1939-1956





Applications in Physics

Fibre bundles with a structural group G are used in electromagnetism and in Yang-Mills gauge theory. G is the gauge group. The gauge potential is represented by a connection, and the gauge field by its curvature. Symmetry-breaking corresponds to a reduction of the structural group G. The Weinberg-Salem model is a gauge theory with symmetry breaking, as well as the standard model

3. From 1939 to 1956

c) Early fifties: Differential Geometry. Local structures

Remains Professor at Strasbourg up to his nomination at Paris in 1955.

He progressively quits Bourbaki ("it took too much time").

.

During this period, several long travels to give courses in Brazil, USA, Iran, India.

1939-1956

Homogeneous spaces (thesis 1934)

Locally homogeneous spaces

Foliated manifolds

Pseudogroups localisation

(Principal) fibre bundles

Connections

Structures

Jets

Reflections on derivatives lead him to introduce the notion of an *infinitesimal jet* to develop a coordinate-free calculus of differentials.

1939-1956

Homogeneous spaces (thesis 1934)

Locally homogeneous spaces

Foliated manifolds

Pseudogroups localisation

(Principal) fibre bundles

Connections

Jets

Two maps f and g from a manifold V to V' have the same k-jet at a point x, denoted by $j_x^k f$ if both have the same partial derivatives at x up to the rank k in a particular (hence in any) coordinate system near x. The k-jets from V to V' form a manifold. In particular the 1-jets from R to V correspond to the tangents and form the *tangent vector bundle* TV of V.

Tangent bundle to the sphere

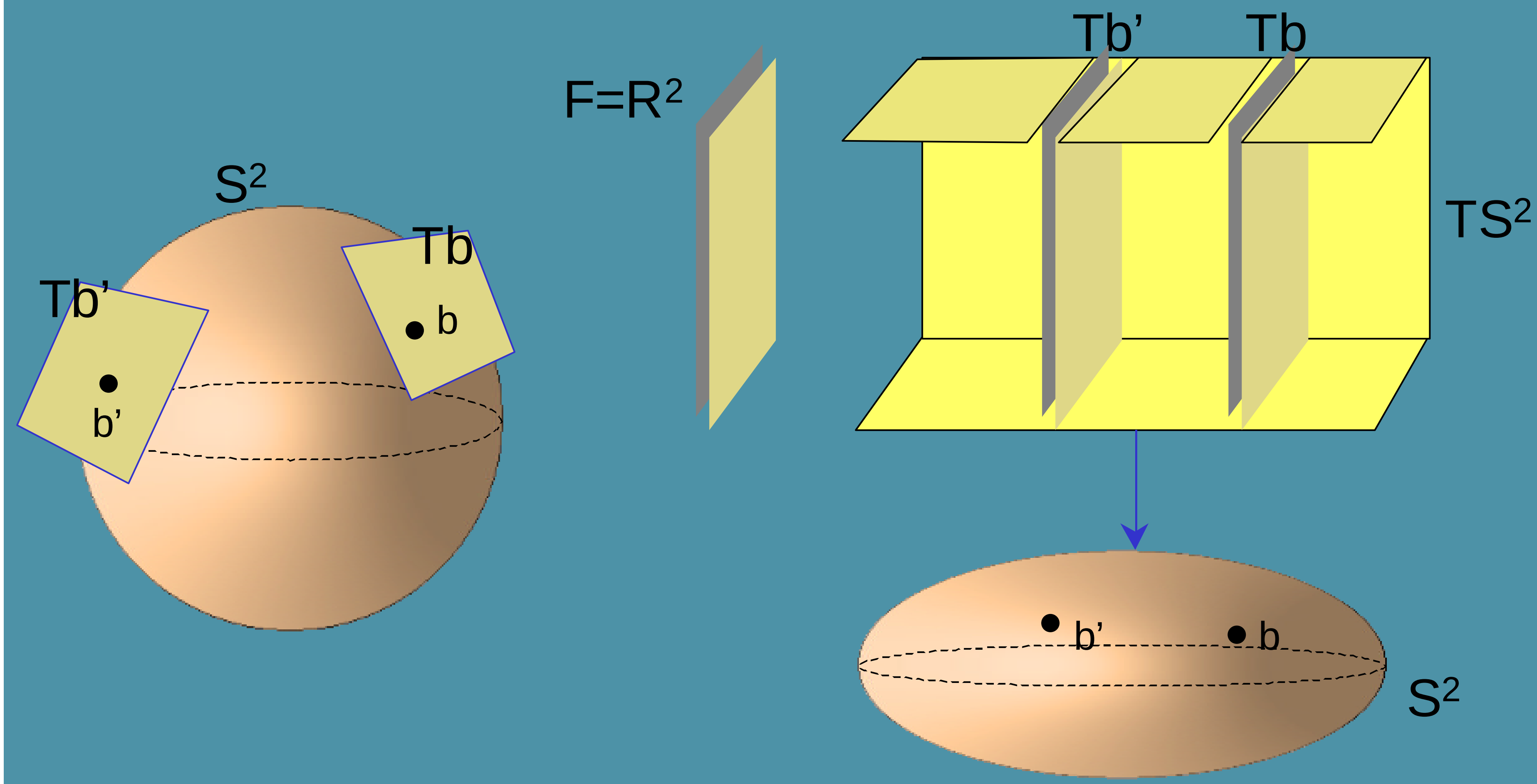


FIGURE 5

Tangent vector bundle to S^2. The base is S^2, the fibre is R^2, the fibre over a point b of S^2 being the tangent bundle at b (formed by the 1-jets at b of maps from R to S^2 representing the tangents at b to the curves drawn on S^2).

1939-1956

Homogeneous spaces (thesis 1934)

Locally homogeneous spaces

Foliated manifolds

Pseudogroups localisation

(Principal) fibre bundles

Connections

Jets

He defines a general composition of jets, which will lead to the "category of jets" (but at that period, he only considers groupoids of jets). He generalizes it to semi-holonomic and non holonomic jets, which intervene in the study of non-integrable differential systems.

1939-1956

Homogeneous spaces (thesis 1934)

Locally homogeneous spaces

Foliated manifolds

Pseudogroups localisation

(Principal) fibre bundles

Connections

Jets

Foundation Differential Geometry

The calculus of jets allows Charles to develop a new foundation of Differential Geometry. It is based on the study of prolongations of a manifold V, defined as the fibre bundles of jets formed by infinitesimal elements of order r, and a main result is the theorem of transitiviy of prolongations.

This approach has been widely adopted and has played a role in the development of *Synthetic Differential Geometry* (Lawvere, Kock,…).

4. The turning-point: 1957-1962

Professor at the University of Paris.

Invited for long stays in Mexico, Buenos-Aires, Sao-Paulo, Montréal.

Our couple is formed during a Congress at Nice in 1957.

He creates the "Cahiers de Topologie et Géométrie Différentielle" in 1958.

In this period, he publishes 3 seminal papers, which are at the basis of his future work.

They constitute a reflection on his former works and a formalization of them in the language of categories.

1957-1962

Categories have been introduced by Eilenberg and Mac Lane in the fourties, but Charles was not familiar with their work at that time.

However in the fifties he had used *groupoids* (defined by Brandt in 1926) which can be seen as categories in which each morphism is invertible.

In particular he had considered the groupoid of isomorphisms between fibres of a fibre bundle, and groupoids of jets

Structures

Pseudogroups localisation

Foundation Differential Geometry

Foliated manifolds

Species of local structures

Foliations

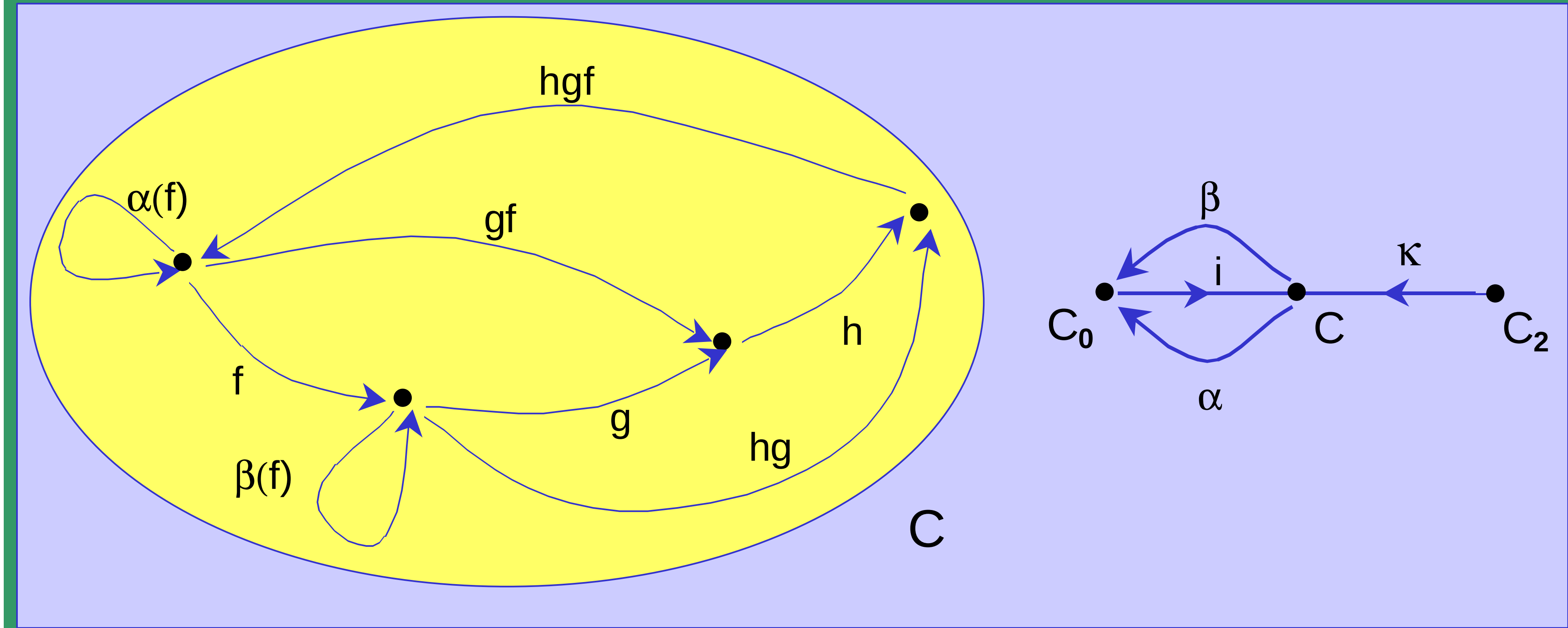
1957-1962

a) 1957: "Gattungen von lokalen Strukturen"

It formalizes the definition of a *local structure*. The main insight is that in a pseudogroup of transformations on D, the "points" of D are not useful, only the open sets of its topology. Thus the topology can be replaced by a "topology without points" (cf. Benabou's talk), a notion which has become extensively used in topos theory in the seventies under the name of "locales".

In the paper, the pseudogroup of transformations is replaced by a local groupoid (or even a *local category*), the associated local structures by a complete *species of local structures*. And the main theorem is the "complete enlargement theorem" which associates to a species of local structures over C a complete species of local structures onto C. (It is a mixture of a general "associated sheaf theorem" and of a Kan extension, at a time when Kan's paper had not yet appeared).

Category and its idea



Let us recall that Charles viewed a category as a (multi-) graph C on which there is given a composition law for 2 successive arrows (f,g) -> gf which is associative and such that each object has an identity. As special cases: group, category associated to an order relation, groupoid, "large" category of sets, or of topologies,….

Category and its idea

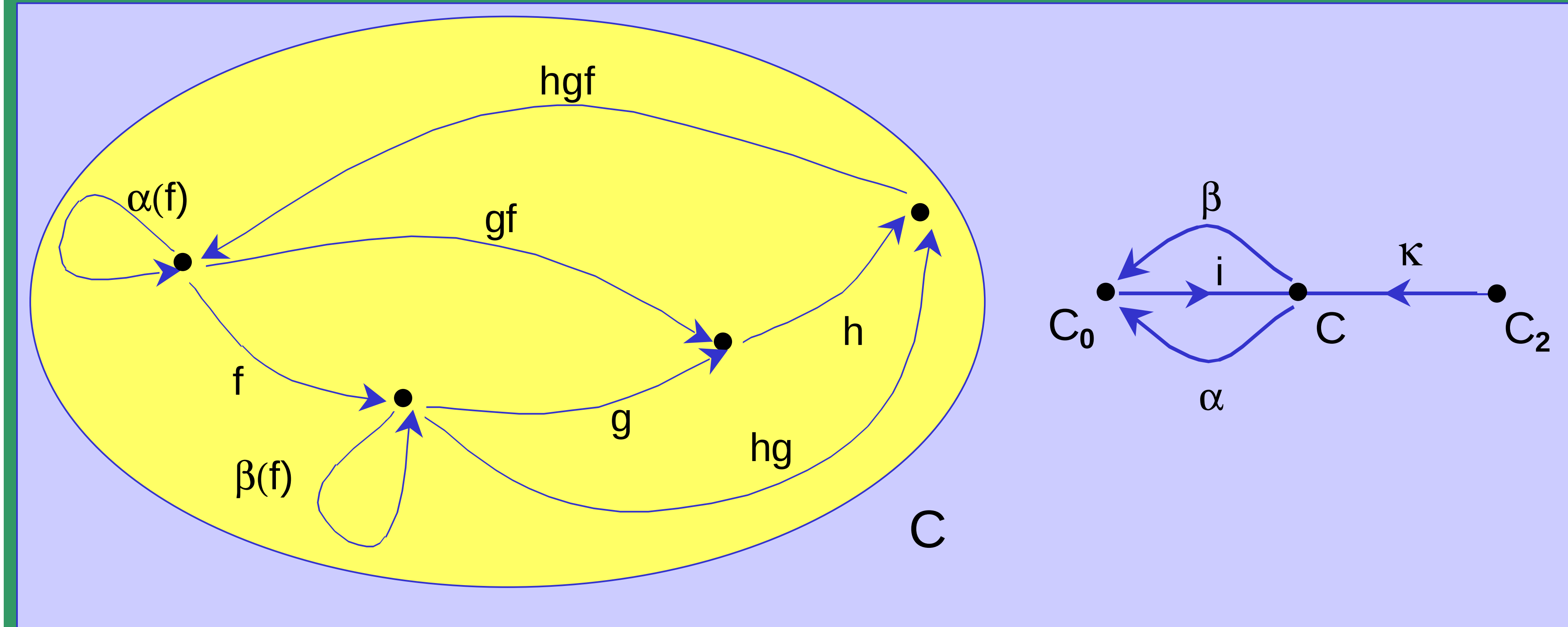


FIGURE 6

C is a category, its objects are the vertices of the graph, its morphisms the arrows f from the source α(f) to the target β(f). There is a composition κ from the set C_2 of 2-paths (f,g) to C which associates a composite gf to (f,g) and satisfies the axioms: associativity: (fg)h = f(gh); identity: f α(f) = f = β(f)f.

Thus the "idea" of the category reduces to the data of the graph formed by α, β, κ.

A species of structure over C

A *species of structures* over C is a set S with a map p from S to the set C_0 of objects of C and a composition (f,s) -> fs iff f in C, s in S and $\alpha(f) = p(s)$ satisfying 2 axioms similar to those for the action of a group on a set (to which it reduces if C is a group). If p is not onto, the enlargement theorem extends the species into a species "onto" C (it is a Kan extension, but before Kan's paper).

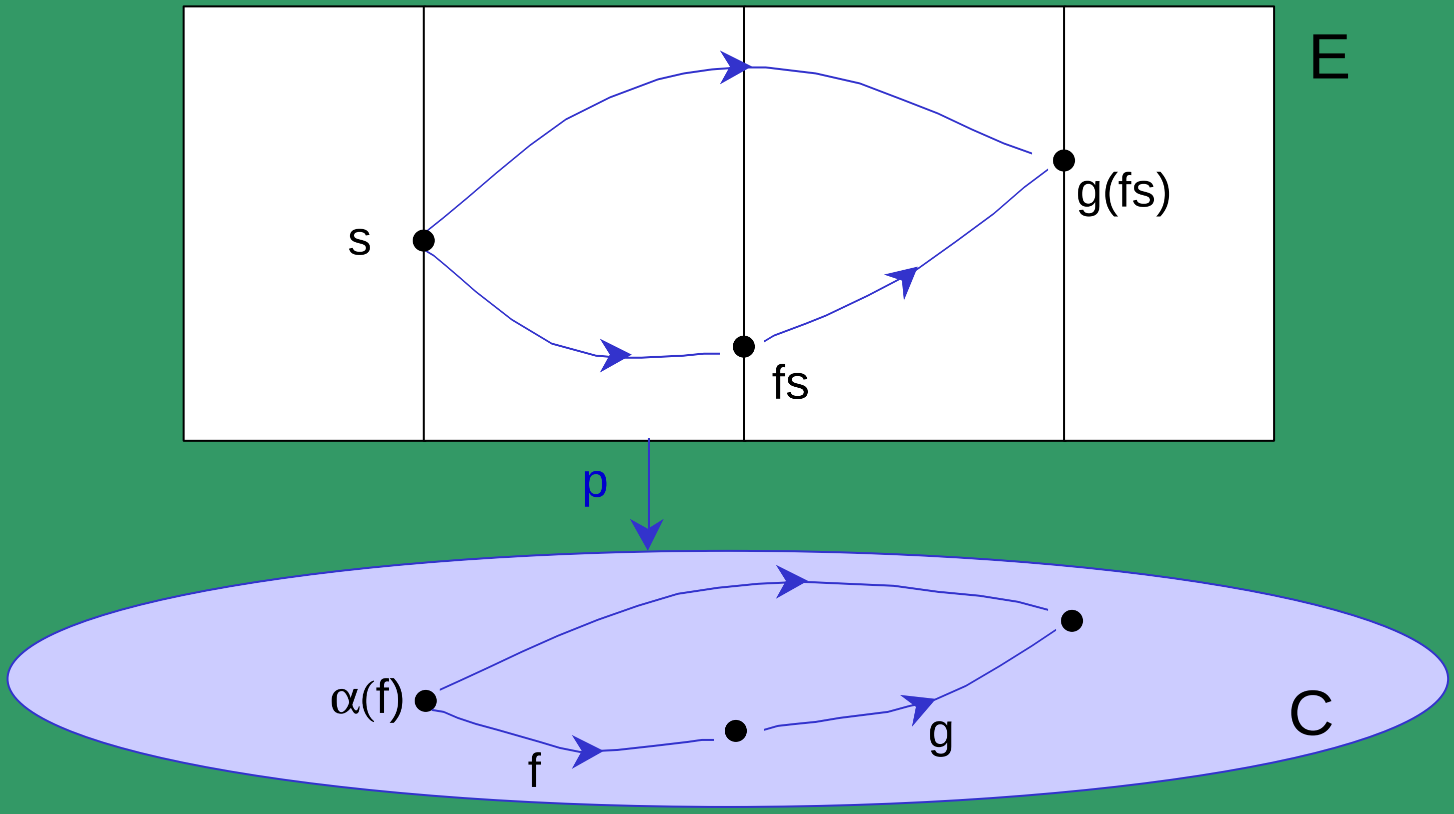


FIGURE 7
The composition of the species S satisfies the axioms: $g(fs) = (gf)s$ and $\alpha(f)s = s$. If p is onto C_0 we have an action of C on S. The idea of the species is formed by the idea of C and p

Structures

Pseudogroups localisation

Foundation Differential Geometry

Foliated manifolds

Species of local structures

Foliations

Ordered categories and species

1957-1962

A local order is an order relation in which each non-void upper bounded part has a least upper bound. A *local category* is a category equipped with a local order compatible with the maps α, β, κ of its idea. Similarly for a *species of local structures.* The species is *complete* if it satisfies a gluing axiom (cf. talk of Guitart); if not it can be made complete by a gluing process (completion theorem).

In a long series of subsequent papers, the local order is replaced by more general orders.

1957-1962

Structures

Pseudogroups localisation → Species of local structures → Ordered categories and species

Foundation Differential Geometry → Topological and differential categories and species

Foliated manifolds → Foliations

b) 1958: "Catégories topologiques et catégories différentiables »

A *topological category* is a category equipped with a topology such that the source, target and composition be continuous. Similarly for a differentiable category, replacing the topology by a differentiable manifold. Idem for a topological or differentiable species of structures, which Charles calls a *generalized fibre bundle.*

This is justified by the fact that the groupoid of isomorphisms between fibres of a principal bundle is topological and acts on the associated bundles. Charles characterizes these groupoids as the *locally trivial groupoids.* He shows that the associated bundles correspond exactly to the topological species on which such a groupoid acts, reducing the theory of fibre bundles to that of topological or differentiable groupoids and their actions.

1957-1962

Structures → Typical functors → Double categories

Pseudogroups localisation → Species of local structures → Ordered categories and species

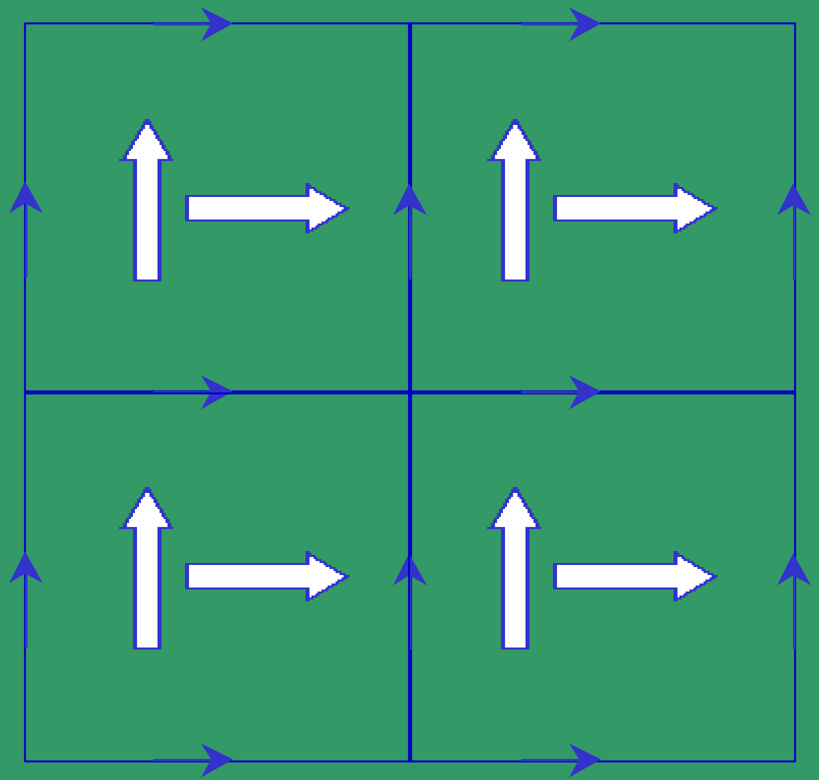
Foundation Differential Geometry → Topological and differential categories and species

Foliated manifolds → Foliations

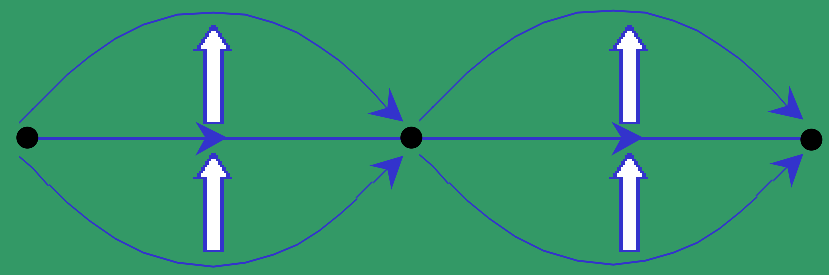
c) Catégorie des foncteurs types

In a 1960 paper (written while we were in Buenos Aires), he formalizes the theory of structures of Bourbaki in the categorical frame, by defining the *typical functors*.

For this, he introduces the notion of a *double category*, considering the 2-categories of natural transformations and some double categories of their squares (which he later called "quintets").



double category



2 category

FIGURE 8

A double category has 2 structures of categories; an element can be represented as a square, and the 'horizontal' and 'vertical' compositions permute so that the figure formed by 4 adjacent squares has a unique composite. It is a 2-category if the objects for the first composition are also objects for the second.

5. From 1962 to 1968: Category Theory

Charles receives the "prix Petit d'Ormoy" from the Académie des Sciences in 1965.

He is made "Docteur Honoris Causa" from the University of Bologna in 1967.

He has several research students, in particular Benabou and Ver Eecke.

Only one long travel: we spent 6 months in Kansas in 1966.

Double categories

Ordered categories and species

Topological and differential categories

p-structured categories and species

1962-1968

a) Structured categories

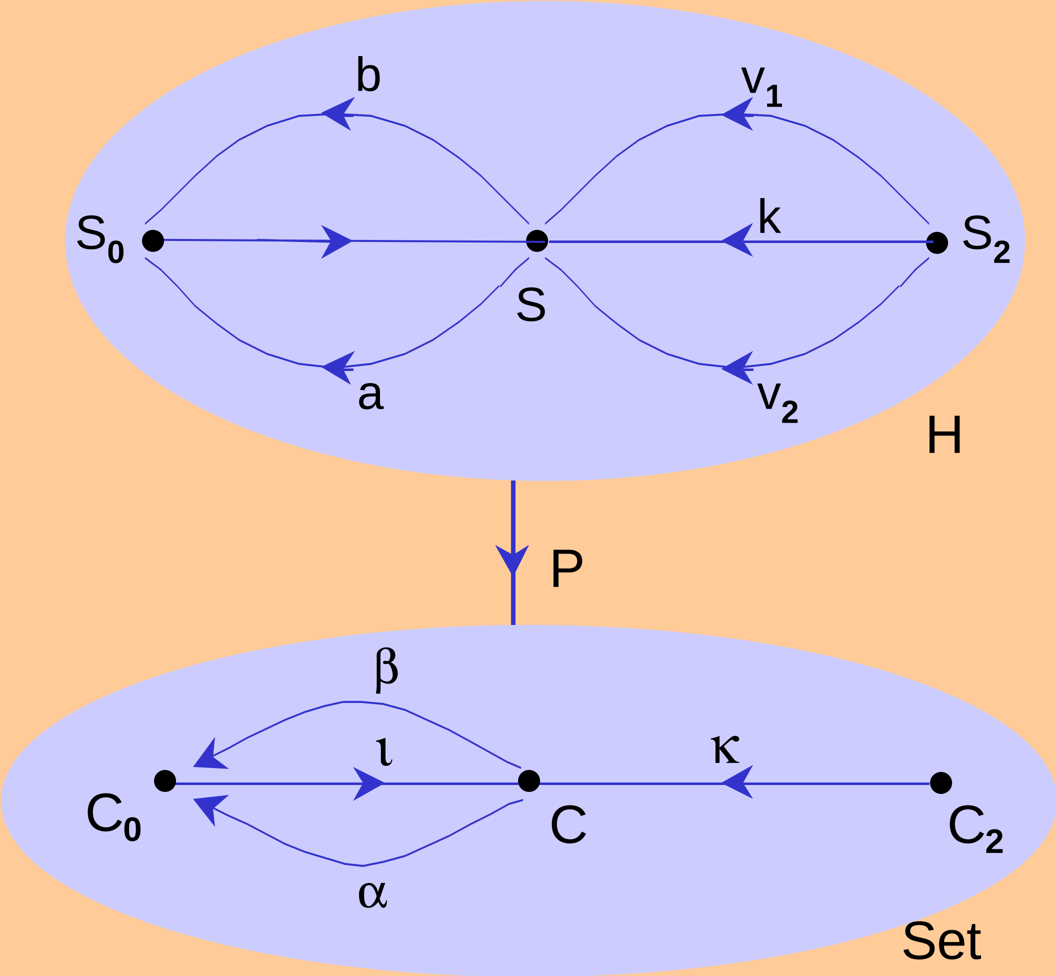
A comparison between the notions of local (or ordered) categories, of differentiable categories and of double categories led us in 1962 to the general notion of a P-*structured category,* where P is a faithful functor from a category H to the category Set of sets; or, in modern terms, internal categories in a concrete category.

The 3 preceding cases correspond to the case where H is a category of ordered spaces, topological spaces, and categories.

A P-structured category (C,s) It is the couple of a category C and an object S of H (the 'structure') on (the set of morphisms of) C such that the maps source, target and composition of C 'lift' in a 'natural' way into morphisms of H.

A P-*structured species of structures* is similarly defined.

P-structured category



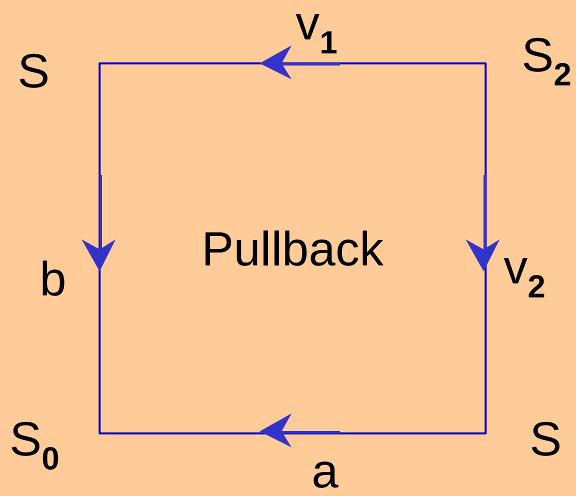


FIGURE 9
C is a category. The maps α, β, κ of its idea lift respectively into morphisms a and b of H from S to a P-sub-object of S and a morphism from the pullback S_2 of (b,a) to S.

Double categories

Ordered categories and species

Topological and differential categories

p-structured categories and species

general theorems depending on various properties of p

1962-1968

General theorems on P-structured categories and P-structured species are given in a series of papers.

They depend on various 'regularity' conditions on the functor P, leading in particular to define sub-spreading functors, or functors with quasi-quotients, as well as some existence theorems for free structures generalizing the usual ones.

1962-1968

Double categories

Ordered categories and species

Topological and differential categories

p-structured categories and species

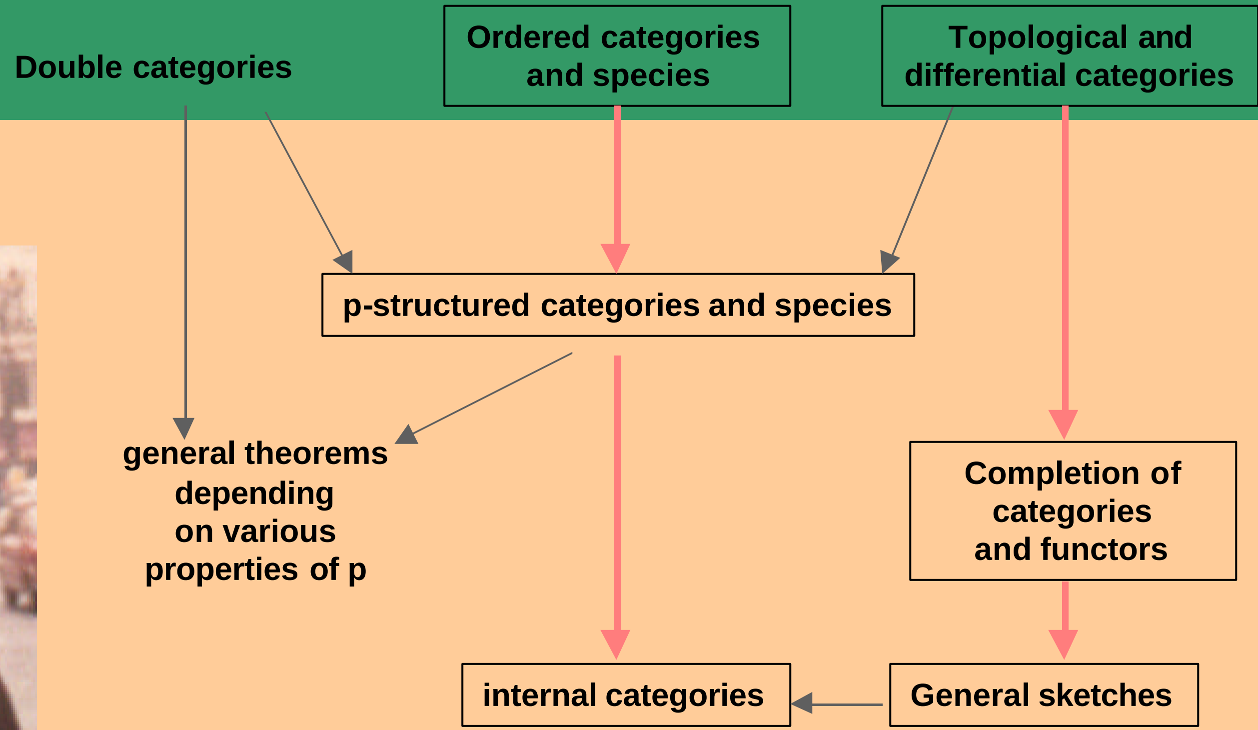
general theorems depending on various properties of p

Completion of categories and functors

b) Completions

When P is not regular enough (as in the case of the forgetful functor from the category of differentiable manifolds), one possibility is to extend it into a good enough functor. Whence a series of extension and completion theorems for categories or functors, also generalizing the complete enlargement theorem for local species of structures.

.

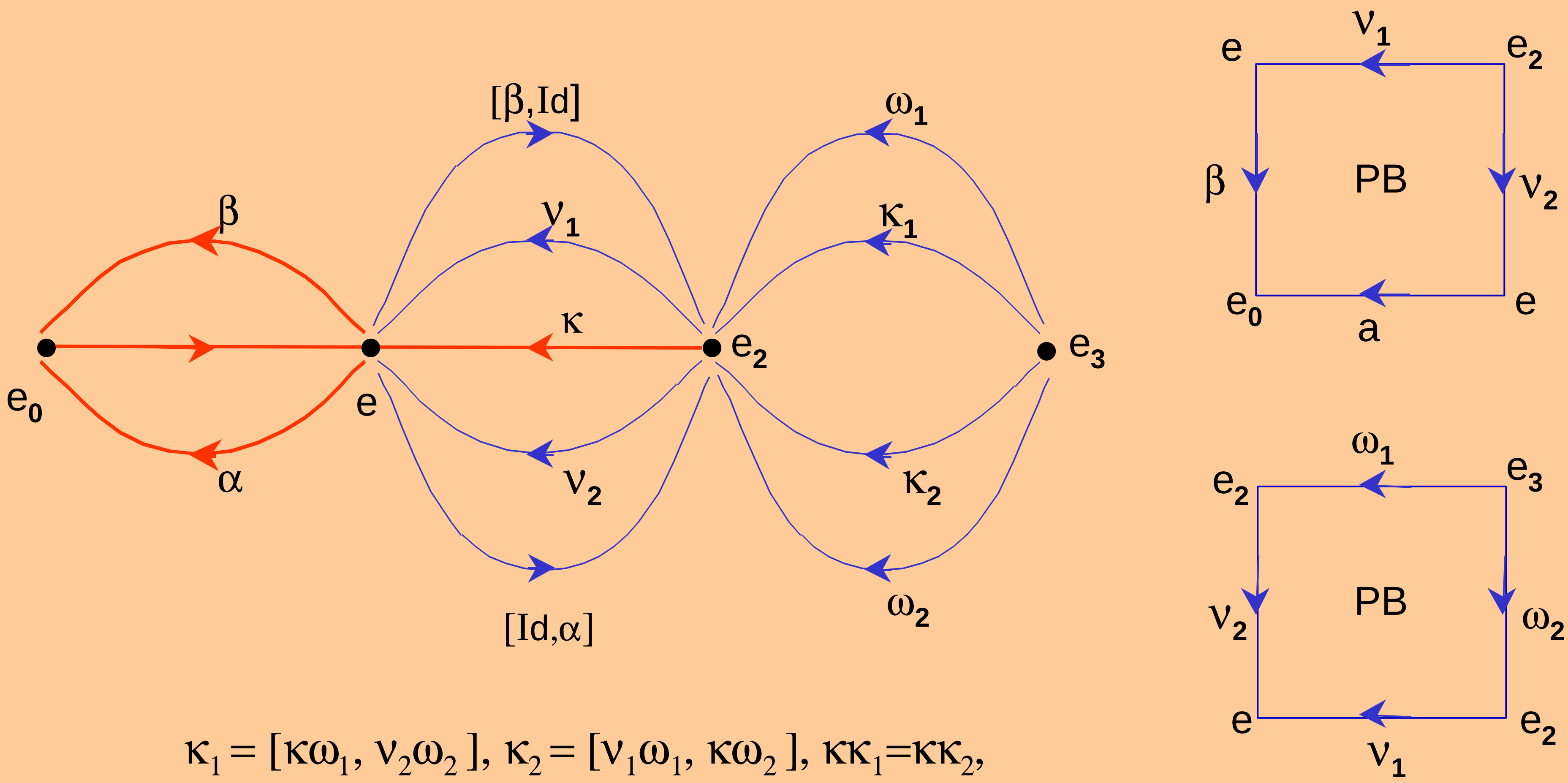


1962-1968

c) Sketches. Internal categories

In the sixties an important subject had been categorical definitions of an algebraic structure (Lawvere, Benabou). Was it possible to do the same for the structure of a category itself? The theory of sketches was introduced to answer this question. In a paper written while we were in Kansas in 1966, Charles introduces a very general notion of sketch. As a particular, we have the *sketch of categories* whose models in Set give the categories and the models in H (above) the P-structured categories. The idea is to add to the maps α, β, κ of the idea well-defined arrows allowing to express the associativity and identity axioms...

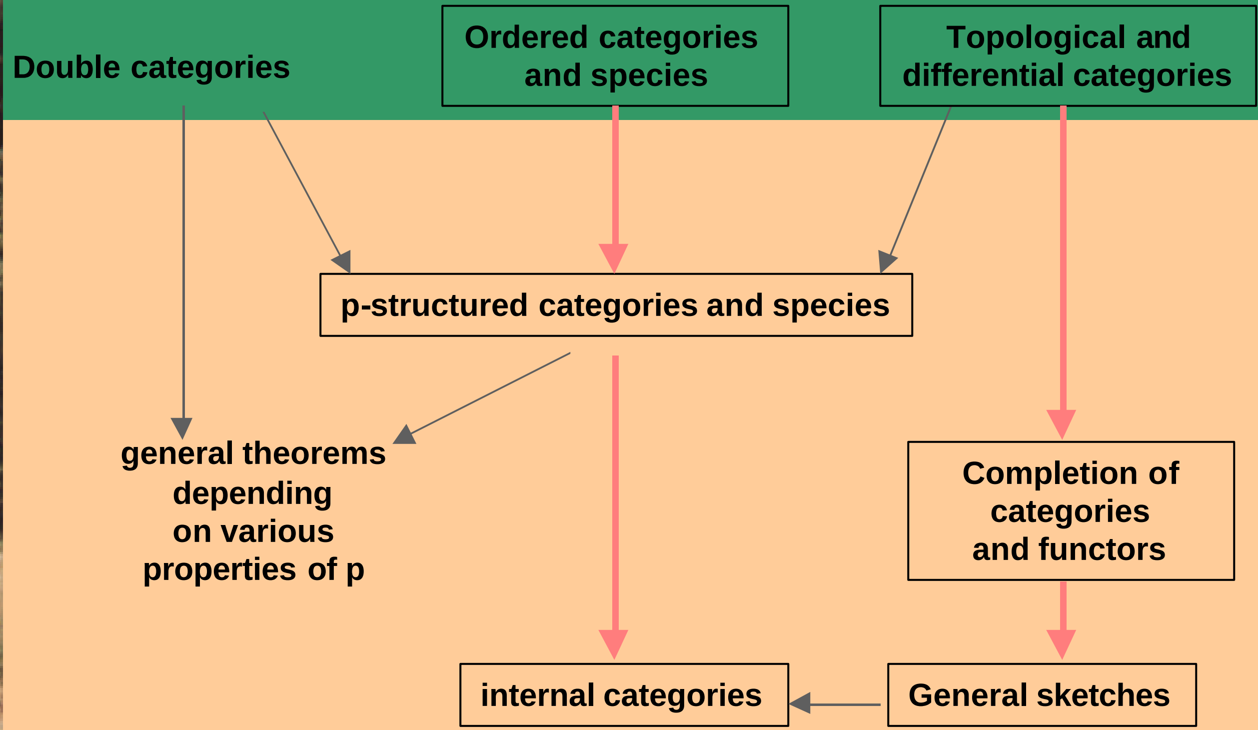
Sketch of categories



$$\kappa_1 = [\kappa\omega_1,\ \nu_2\omega_2\,],\ \kappa_2 = [\nu_1\omega_1,\ \kappa\omega_2\,],\ \kappa\kappa_1{=}\kappa\kappa_2,$$

FIGURE 10
The sketch of categories is the following category equipped with 2 cones to become pullbacks in the models. (It is a full sub-category of the opposite of the simplicial category)

1962-1968



A model of the sketch of categories in any (not necessarily concrete) category K is called by Charles a *generalized structured category*, now an *internal category* in K. (Another definition of internal category had been given by Benabou in 1964.)

6. From 1969 to the end: Sketched structures

We live in Paris up to 1975, when Charles had to retire from Paris.

His last years there were marred by some hostility against category theory in general, and our research team in particular.

Then we settled in Amiens, where he gave courses and actively participated to the life of the University, being adjoint Director of the Faculte de Mathématique and Director of its Conseil Scientifique.

6. From 1969 to the end: Sketched structures

During this period, a large part of our time is devoted to our research team "Théorie et Applications des Catégories, Paris-Amiens".

7 "doctorats d'Etat" and about 40 "doctorats de 3e cycle" are prepared in it.

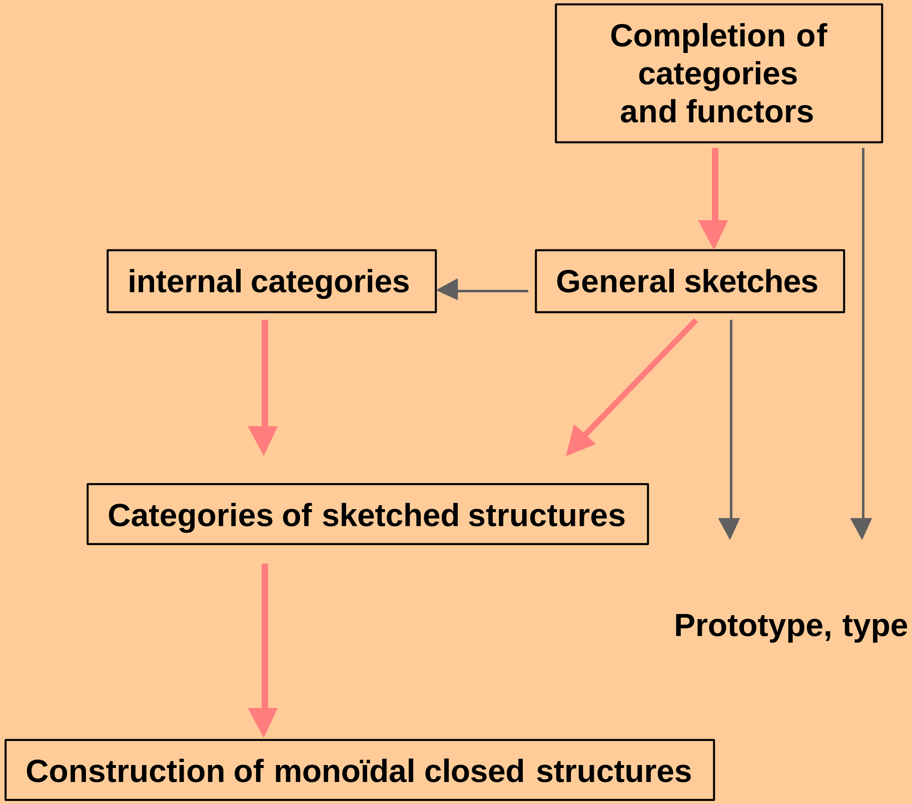
We organize 3 inter-

national conferences

and monthly

"Journées TAC".

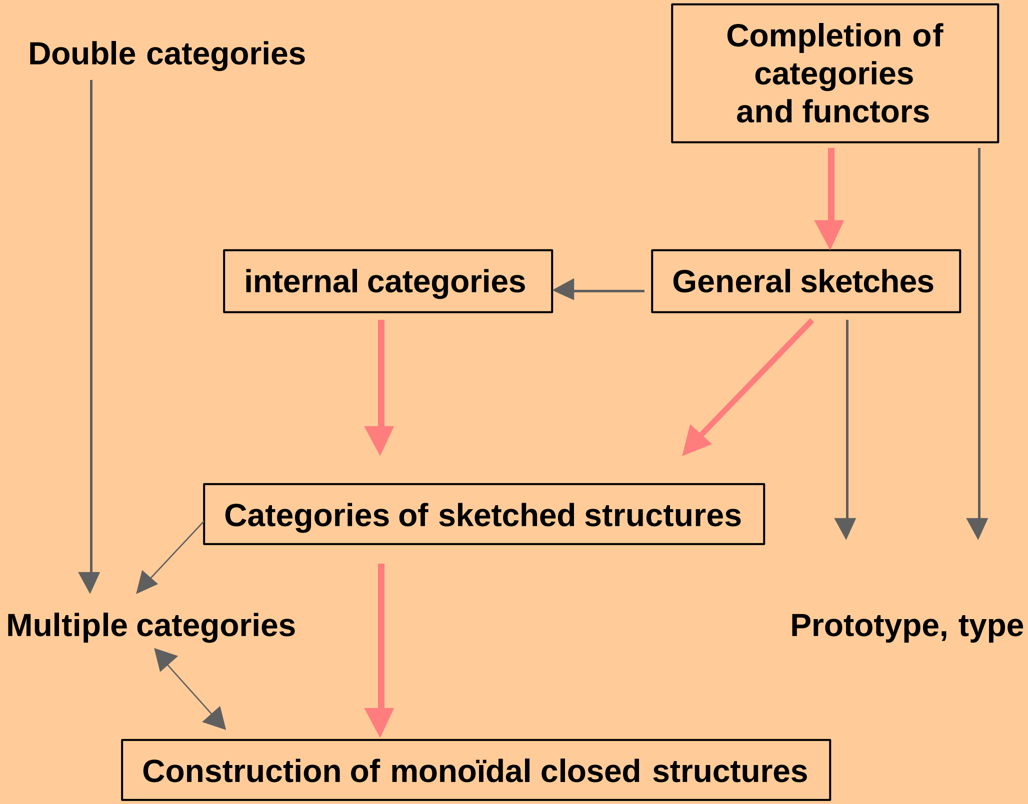
1969 -1979



a) Sketch theory

To obtain general theorems, the notion of a sketch must be restricted to that of a category with (inductive and projective) cones, or even just projective cones.
A given structure can be represented by several sketches. In a long paper (1972) we complete a sketch into *a prototype* or a *type* to get a more complete representation.
In the same paper, we study categories of sketched structures, in the case of projective sketches (Guitart and Lair later generalized this to mixed sketches), and describe monoidal closed structures on them.

1969 -1979



b) Multiple categories

The last series of papers we wrote jointly concerns the theory of *multiple categories*. It was terminated when we were in the hospital during the last illness of Charles (due to kidney failure). We describe two different closures on the category of multiple categories.

Charles died at home the 24 September 1979.

After his death I edited his complete works (7 volumes) adding numerous comments to clarify the terminology and add more recent developments.

The first volume was published in 1980, just before the international Conference dedicated to Charles I organized in Amiens.

Later on, the University decided to give his name to the amphitheatre in which to-day we commemorate his 100th birthday.