INTRODUCING ABSTRACT MATTER[1]

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1. INTRODUCTION

In this paper we will introduce a new kind of structure which we will call Abstract Matter (AM). It is an abstract mathematical structure which in many ways will reflect and model the way in which matter — both organic and inorganic - is being built. It is the starting point of a general theory of structure and organization which will be useful in designing models and experiments of how to synthesize various kinds of matter.

Abstract Matter is an extension and explication of our previous notion of a hyperstructure for which we refer to [1 - 7]. In 1995 we introduced in [5, 6, 7] the notion of Categorical Matter. This previous notion of Categorical Matter is included in the new notion of Abstract Matter.

Hyperstructures were introduced as a framework to combine hierarchies, higher order structures and emergence in a general way in order to include example from physics, biology and other fields as well. In mathematics higher order structures have mostly been studied in special forms in logic and set theory. At the time we were developing the notion of hyperstructures, the mathematical notion of higher categories (n-categories) was revived and a new and very extensive development started [11, 13, 15, 16]. Our study and use of higher categories in purely mathematical research [9] has been another motivation for introducing a somewhat more general and relaxed structure like Abstract Matter.

What is the idea?

Any kind of complex system or structure is built up from some elementary pieces and constructed layer by layer. The problem is to find a good framework for describing this in a non-trivial way. In our opinion a good definition is the clue to further progress in our understanding of such systems. In the present approach we will emphasize the notion of bonds, interactions, relations and relationship at one level and investigate how new levels and higher order structures are being created. The next challenge is then to construct and develop detailed examples of Hyperstructures, Abstract Matter and Dynamical Hierarchies with the present definitions and framework as guidelines.

[1] The present paper is under revision and extension

2. HYPERSTRUCTURES

We will here present a new version of hyperstructures and this will be done in several steps.

Construction 1. We start with a set of objects X_0 — our basic units.

To each subset

$S_0 \subset X_0$

we assign a set of properties or states, $\Omega_0(S_0)$, so

$$\Omega_0 : \mathcal{P}(X_0) \to \text{Sets}$$

where $\mathcal{P}(X) = \{A | A \subset X\}$ — the set of subsets — the power set, and Sets denote a suitable set of sets. (In the language of category theory $\mathcal{P}(X_0)$ would be considered as a category of subsets, Sets - some category of sets.)

Then we want to assign a set of bonds, relations, relationship or interactions of each subset S_0 — $B_0(S_0)$ — depending on properties and states. In this paper we will just call them bonds. Let us define

$$\Gamma_0 = \{(S_0, \omega_0) | S_0 \in \mathcal{P}(X_0),\ \omega_0 \in \Omega_0(S_0)\}$$

$$B_0 : \Gamma_0 \longrightarrow \text{Sets}$$

In our previous notion of hyperstructures the set X_0 represents the systems or agents $(S_i), \Omega_0$ the observables (Obs), B_0 the interactions (Int) and a specific choice of $b_0(S_0) \in B_0(S_0)$ represents the resultant "bond" system giving rise to the next level of objects — called R in previous papers, like S_i, Obs, Int, see [1-7].

In the case of categories there are no states and S_0 consists of two ordered elements (X, Y). The bond set $B(S_0) = B(X, Y)$ is then the same as the set of morphisms $\text{Mor}(X, Y)$, see the Mathematical Appendix.

Now let us form the next level and define:

$$X_1 = \{b_0 |\, b_0 \in B_0(S_0, \omega_0),\ S_0 \in \mathcal{P}(X_0),\ \omega_0 \in \Omega_0(S_0)\}$$

by definition the image set of B_0, and

$$\begin{array}{c} X_1 \\ \downarrow \pi_0 \\ \mathcal{P}(X_0) \end{array}$$

$\pi_0(b_0) = S_0$

X_1 represents the bonds of collections of elements or interactions in a dynamical context. But the bonds come along with the collection they bind like morphisms in mathematics come along with sources and targets. Similarly at this level we introduce properties and state spaces and sets of bonds as follows:

$$\Omega_1 : \mathcal{P}(X_1) \longrightarrow \text{ Sets}$$

$$\Gamma_1 = \{(S_1, \omega_1) | S_1 \in \mathcal{P}(X_1), \omega_1 \in \Omega_1(S_1)\}$$

$$B_1 : \Gamma_1 \longrightarrow \text{ Sets}$$

Then we form the next level set:

$$X_2 = \{b_1 | b_1 \in B_1(S_1, \omega_1),\ S_1 \in \mathcal{P}(X_1),\ \omega_1 \in \Omega_1(S_1)\}$$

and

$$\begin{array}{c} X_2 \\ \downarrow \pi_1 \\ \mathcal{P}(X_1) \end{array}$$

$\pi_1(b_1) = S_1$.

We now continue iterating this procedure up to a general level N:

$\Omega_{N-1} : \mathcal{P}(X_{N-1}) \rightarrow$ Sets
$B_{N-1} : \Gamma_{N-1} \rightarrow$ Sets
$X_N = \{b_{N-1} | b_{N-1} \in B_{N-1}(S_{N-1}, \omega_{N-1}),\ S_{N-1} \in \mathcal{P}(X_{N-1}),\ \omega_{N-1} \in \Omega_{N-1}(S_{N-1})\}$

This is not a recursive procedure since at each level new assignments take place. The higher order bonds extend the notion of higher morphisms in higher categories.

Let us write
$\mathcal{X} = \{X_0, \cdots, X_N\}$
$\Omega = \{\Omega_0, \cdots, \Omega_{N-1}\}$
$B = \{B_0, \cdots, B_{N-1}\}$

Further mathematical properties to be satisfied will be discussed elsewhere.

Definition: The system $\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B})$ where the elements are related as described, we call *a hyperstructure of order N*.

Construction 2.
In construction 1 we considered arbitrary subsets $S_{j(i)} \subset X_i$ and assigned $\Omega_i(S_{j(i)})$ and B_i to these. But in many situations one may want to consider collections of special types of subsets like:
finite subsets
ordered subsets
simplicial subsets
families of indexed sets
etc.

Such collections may easily be obtained from the full subset collection by bond-type structures. These bond-type structures we just suppress in order not to make the construction too complicated and hiding the basic idea.

In conclusion we could pass from arbitrary subsets to collections of "structured" subsets:

$$\mathcal{P}(X_i) \rightarrow \text{Coll}(X_i)$$

and then define Ω_i and B_i on these.

This is just a useful refinement to be aware of.

Construction 3

There is another important extension of the fundamental idea in the previous constructions.

The sets representing the objects of the various level were $X_0, X_1, \cdots, X_N$. The elements of these sets had no internal structure. However, we will see how this situation fits into a more general construction scheme.

Let us be given a sequence of families of structure types.

$$\mathcal{S} : \mathcal{S}_0,\ \mathcal{S}_1, \cdots, \mathcal{S}_n,$$

The structure types could be of mathematical, physical, biological etc. type depending on the context. Examples could be: categories, algebras, atoms, molecules, cells, organisms, etc. The mathematical ones would include Bourbaki's structures and species of structures, see [12]. At this stage we will not be more specific regarding structure types.

Extension:

The set X_0 could be a set or family of structures of a type in the collection $\mathcal{S}_0$.

Furthermore X_0 could also itself be organized into one of these structure types. The property and state assignment Ω_0 may not just take values in sets, but in sets with additional structure of type in $\mathcal{S}_0$ (for example categories).

However, the bond-assignment is lifting us up to a higher level where we require that

$$B_0(S_0, \omega_0)$$

is of structural type in $\mathcal{S}_1$

$$X_1 = \{b_0 |\ b_0 \in B_0(S_0, \omega_0),\ S_0 \in \mathcal{P}(X_0),\ \omega_0 \in \Omega_0(S_0)\}$$

is then a set of structures of type in $\mathcal{S}_1$ and may also itself be structured by one of these types. We then continue in this way and end up with X_N of structure type in $\mathcal{S}_N$. In this way we obtain new structure types — hyperstructures of order 1, 2,... and these may be added to the given structure types and hence play a role in the further level constructions.

We include this in

Definition:

$\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B}, \mathcal{S})$

we call a hyperstructure of order N and structure type $\mathcal{S}$.

This gives the design of the framework of hyperstructures, in essence it is the architecture of general structure and organization. In addition there are mathematical and other context sensitive conditions to be satisfied. We will return to this later.

Examples are hyperstructures in the old sense [1], higher categories (n-categories, n-fold categories), multilevel systems like biological structures bound together level by level — for example molecules, cells, tissues, organs, organisms. More examples and details will be specified and given elsewhere, but we will end with a geometric and topological example which illustrates the intuition behind the present structure.

Example

In geometry and topology we consider a kind of generalized surfaces in arbitrary dimensions called manifolds. These may be smooth and have various additional structures. Among manifolds there is a very important notion of cobordism:

Two manifolds A and B of dimension n are cobordant iff there exists an $(n+1)$ dimensional manifold C such that

$$\partial C = A + B, \quad \text{(disjoint union)}$$

∂ stands for boundary and we ignore orientation here. A and B may consist of several components. C is called a cobordism between A and B, and may in our terminology be thought of as a bond of $A+B$ or their components, see Figure 1.

In this paper we are interested in structures of structures etc., so what about cobordisms of cobordisms or more generally: bonds of bonds in their geometric situation?

Let C_1 be a bond (cobordism) between A_1 and B_1, and C_2 correspondingly between A_2 and B_2. Then D is a bond of C_1 and C_2 iff

$$\partial D = (C_1 + C_2) \cup (\hat{C}_1 + \hat{C}_2)$$

where $\cup$ means glued together along common boundary: $\partial(C_1 + C_2) = \partial(\hat{C}_1 + \hat{C}_2)$ and D of dimension $n+1$, see Figure 2.

Furthermore a third order bond between D_1 and D_2 would be given by an $(n+2)$ dimensional manifold E such that

$$\partial E = (D_1 + D_2) \cup (\hat{D}_1 + \hat{D}_2)$$

etc, see Figure 3.

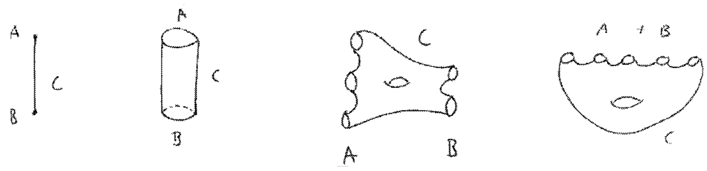


Figure 1.

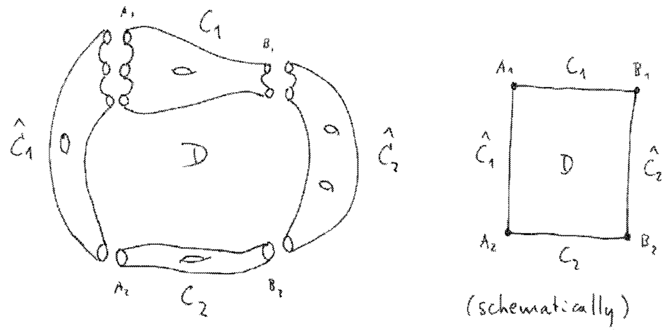
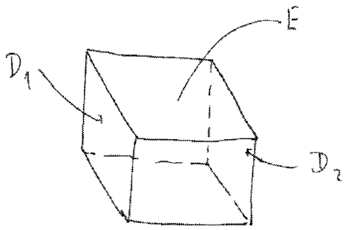


Figure 2



E is schematically represented by a cube.

Figure 3.

In the framework we have introduced these geometric examples in the figures correspond to:

$$X_0 = \{ \text{ the set of circles in some high dimensional space}\}$$

no states, $\Omega_0 = \emptyset$.

$S_0 \in \mathcal{P}(X_0)$ means that S_0 is a disjoint union of circles. $b_0 \in B(S_0)$ is then given by a surface having the circles of S_0 as its boundary

$$X_1 = \{\text{the set of surfaces with boundary equal the union of circles}\}$$

$S_1 \in \mathcal{P}(X_1)$ is a disjoint union of such surfaces.

$B_1(S_1)$ is then given by a 3 dimensional manifold having the surfaces of S_1 as parts of its boundary, but possibly glued together along common boundaries with additional parts - the $\hat{C}$'s.

And in this way it goes on up to a desired dimension. If in addition we may add states in the form of letting the Ω_i's take vector spaces (Hilbert spaces) as values we enter the situation of topological quantum field theory which we will not pursue here.

3. ABSTRACT MATTER

Our point of view is that hyperstructures represent an efficient and useful way of describing the basic structure of complex matter — both organic and inorganic, as well as other organizational structures. Let X be the set of basic elements, building blocks or agents in a certain context (the "atoms" or "elementary particles" of the situation). Then we put

$$X = X_0$$

and proceed by developing a hyperstructure $\mathcal{H}(X)$ of order N.

Definition. A hyperstructure $\mathcal{H}(X)$ of order N on a basic set or structure X is called a piece of Abstract Matter of order N.

We may have situations where at each level only some of the possible bonds, states and properties are being taken into account or realized. Such a structure we call a substructure or an instance of the actual hyperstructure $\mathcal{H}(X)$ — but considered a piece of abstract matter. X may already have a structure like that of a group, algebra, manifold, etc.

Abstract matter represent entities organized in a specific way. Intuitively one may think of the bonds as forming a kind of "membranes" around the entities they are binding. The higher bonds then form "membranes" around the previously formed "membrane" systems taking into account acquired and observed properties and states, see the following figure 1.

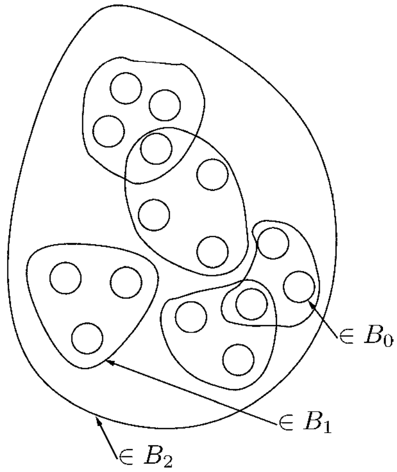


Figure 4.

Let us again emphasize here that we think of bonds representing: binding mechanisms, interactions, relations, relationships, connections, etc. The point is to find a framework for their higher order versions. Abstract Matter should provide a useful modelling and simulation tool for a series of physical, chemical, biological and social and economical systems. The basic idea being that they are all governed by a bond-type structure as described in the definition of hyperstructures. In addition one should also note that advanced technological systems (cars, computers, etc.) are non-trivial higher order bond systems. But the nature of the hyperstructures in the different examples will reflect their special properties. The hyperstructures in physics will depend on the laws of physics likewise in biology, etc. But in other Abstract Matter one may of course introduce freely one's favorite rules and laws.

4. DYNAMICS

In a hyperstructure $\mathcal{H}$ of order N the "global" type elements are represented as bonds b_N in

$$B_N(S_N, \omega_N)$$

and at each level we have families of bonds and states. These may also depend on time:

bonds: $b_N(t), \cdots, b_i(t), \cdots, b_0(t)$
states: $\omega_N(t), \cdots, \omega_i(t), \cdots, \omega_0(t)$.

When such time dependence is present, it gives rise to dynamics of hyperstructures and abstract matter. Whether the dynamics is discrete or continuous does not matter from the point of view of abstract matter, the principles are the same. So we consider here the discrete case.

Definition.
A dynamics on a hyperstructure $\mathcal{H}$ with time dependent bonds and states is

given by families of rules

$$\mathcal{R} = \{R_i\}$$

where

$$R_i : \begin{array}{l} b_i(t) \to b_i(t+1) \\ \omega_i(t) \to \omega_i(t+1) \end{array}$$

in such a way that the change of bonds and states are compatible with the general bond and state-structure ($\mathcal{B}$ and Ω). This is the basic principle for updating the system. Even the rules could be organized as hyperstructures, and hence be of AM-type, but we will not pursue this aspect here.

One will often search for abstract matter represented by stable equilibria or fixed points, in other cases one may want periodic cycles to subsist in the hyperstructure at one or several levels at the same time. General dynamical systems of the input/output type may be considered as a kind of reactors (or processes). The "reactors" may just be thought of as bonds between inputs and outputs. In our setting of hyperstructures and abstract matter we are then able to discuss higher order dynamics like reactors of reactors,...., processes of processes,...., dynamics of dynamics......

Therefore in our view the ultimate notion of a Dynamical Hierarchy is a hyperstructure with a dynamics. But hyperstructures are more profound than hierarchies since the properties or states (Observer mechanisms) of lower levels are determining factors of the higher levels and the emphasis is on bonds rather than objects.

Our basic illustration and examples of dynamics in this paper is Higher Order Cellular Automata in section 7, with a detailed discussion in [10].

5. DESIGN AND MODELLING

What is the purpose of general schemes like hyperstructures and abstract matter? In mathematics, physics, chemistry, biology and many other sciences one studies systems of great complexity and at several levels. In order to understand, study and synthesize such systems it is important to have abstract models exhibiting their essential features. This is precisely the purpose of hyperstructures and abstract matter, namely to provide a general construction and design principle of models and synthetic systems, emphasizing the essential features. This will facilitate computer simulations of real systems of complex matter like new materials and biological structures. The idea is that extracting the basic features of some systems and formalizing them in Abstract Matter one should be able to make predictions about real matter.

In the artificial world or universe of Abstract Matter one can then freely regulate the rules and laws. But also the laws, prescriptions, rules, processes, programs, dynamics, etc. in these abstract worlds will in general be hyperstructural and hence pieces of Abstract Matter as well ! The idea is that all kinds of matter and energy are in some way built up according to a hyperstructure principle. Hence our world — both physical and mental — is governed by hyperstructures realized as Abstract Matter. The important thing is to identify or introduce the relevant hyperstructures in order to manipulate both the real and abstract worlds.

In the picture of AM reactors production of AM may require AM descriptions and processes. For example something analogous to the photosynthetic process in biology — possibly in simplified form may be needed. Also genetic instructions in the genome and DNA should be viewed as AM and thought processes and consciousness as "mental" AM. One may use DNA-type AM to code not only for biological instructions, but general "materials". The new "genome" in this case being a piece of AM in the sense described.

When one describes the construction of a piece of AM, the description should itself be a piece of AM — like DNA. This should then be put into a "seed" of AM in order to produce the piece of AM in a reactor. Such production seeds of AM generalize basically the genome.

The reactor needs AM building blocks and "energy" via rules and dynamics. It seems like an interesting task via hyperstructured AM to give a "genetic" production description (of AM type) of all kinds of materials and matter.

6. MATHEMATICAL ASPECTS

A mathematical theory of hyperstructure should be developed — a daunting task. Hyperstructures extend n-categories, n-fold categories and multisimplicial sets. For n-categories the general theory is now being rapidly developed and applied both to mathematics and physics. This theory gives some indications of where to go, but also totally new aspects appear in our context. Certainly a calculus of bonds and their compositions should be developed specifying the rules and equations or relations they should obey. The bond structure we have introduced loosens up the morphism structure in higher categories and this may give several advantages. One may for example study geometric bonds without having sources and targets and this will be developed elsewhere. Geometric glueing is important in geometric topology and may be considered as a bond structure in our sense. The higher order glueing which is necessary in Topological Quantum Field Theory [11] is well taken care of in the hyperstructure context, see the geometric example in 1.

In mathematics as in other fields it is important to have a framework for forming totalities of totalities, etc. For example the totality of all surfaces of a certain kind are parametrized or represented by points in another space — commonly called the moduli space. This is a deep construction and the study of moduli spaces is a profound piece of mathematics.

Hyperstructures actually do the same thing, forming totalities via bond structures and observed states and properties, which again are organized into new totalities. A totality is a bond-mechanism of its constituents and vice versa. From a mathematical point of view it is interesting to think of hyperstructures as higher order moduli-structures. This means that we have a family of structures which we parametrize or organize into another kind of structure, a moduli structure being a hyperstructure of order 1. Then different such structures may again be parametrized or organized into another kind of structure, a second-order moduli-structure which is then a hyperstructure of order 2. So this goes on to moduli-structures of order N.

In the same way as we introduced dynamics on hyperstructures we can now introduce dynamics on higher order moduli structures. This seems quite interesting looking at some examples coming from 2-level structures:

i) Dynamics of dynamics, correspond to "renormalization" in physics and dynamics and the second order dynamics - renormalization - represent interesting universal dynamics (Feigenbaum map, etc.)
ii) Renormalization flow on the "Space of quantum field theories". Here the fixed points are conformal field theories.
iii) In the subtle study of geometries of 3-manifolds all the Thurston-geometries appear as flow fixed points [14].

This is just meant to illustrate the versatility of the hyperstructure idea, but it also illustrates the idea of constructing AM with special desired properties as for example fixed points of dynamics at a higher level.

In addition to develop a calculus of bonds one should classify hyperstructures, study dynamics and their fixed points, cycles, attractors etc. There seems to be an ocean of interesting unexplored territory.

7. HIGHER ORDER CELLULAR AUTOMATA

In [10] we introduce the notion Higher Order Cellular Automata along the lines of hyperstructures. So this is a concrete example of this kind of higher order structures. We carefully and detailly introduce second order cellular automata (2-CA). In 2-CA we "bind" cells to families of cells which we call organs. In our terminology an organ is a bond structure of its cells. This means that B_0 picks out the subsets of cells (S) forming aggregates called organs. Therefore

$$B_0(S) = \{S\} \text{ if } S \text{ is an organ}$$

and

$$B_0(S) = \phi = \text{ the empty set if } S \text{ is not an organ.}$$

In the simplest case here $\Omega_0(S) = \phi$.

In the same way states bind to families of states, neighbourhoods to families of neighbourhoods and rules to families of rules. These bonds or totalities then represent the higher order notion. The dynamics involves all levels. 2-CA are important examples of dynamical hierarchies.

Explicit examples are given in [10] and 2-CA show new and interesting dynamical behavior, and certainly deserve further attention. The same ideas may easily be applied to more general graph-automata and network-dynamics. We think that second order cellular automata (2-CA) and the development of second order K-theory (2-K), see [9], are interesting mathematical examples showing the potential for novelty in hyperstructure type constructions. Hence we conclude in (10): Take your favorite concept and put 2 in front of it and see what happens. If something new and interesting shows up, continue!

8. HIGHER ORDER UNIVERSES

In science one often constructs various kinds of universes or worlds — real or abstract — in order to study certain phenomena.

This could be universes of sets, categories, vector spaces, atoms, molecules, cells, societies, etc. Often new situations arise when families of universes occur (sets of sets, categories of categories, societies of societies, etc.). Then we are in the setting of forming hyperstructures and Abstract Matter as described, and they form second order universes, third order universes, etc. Also concepts lifts from lower order universes to higher order universes.

Our general claim is that these new universes — when introduced correctly — are useful and interesting objects of study bringing forward genuinely new properties.

9. CONCLUSION

We have here introduced a revised form of hyperstructures more suitable for both theory and applications. It clearly shows what in our opinion is the basic structure of all types of dynamical hierarchies. This is just meant to be the beginning of a vast new area of research, where already some examples show profoundly new results. The notion of Abstract Matter should be useful in performing computer experiments and simulations of many complex real world systems. It gives a freedom and flexibility which is not present in ordinary matter and hence more easy to manipulate for desired purposes. In addition it is an ideal universe for creative experimentation and design.

Finally be aware of: outside any 1-world (or universe) there is a 2-world, a 3-world and an N-world and somewhere an ∞-world.

References

(1) N.A. Baas, Emergence, Hierarchies, and Hyperstructures, Artificial Life III, Santa Fe Institute Studies in the Sciences of Complexity, ed. C.G. Langton, Addison-Wesley, 1994 (515–537).

(2) N.A. Baas, Hyperstructures as Tools in Nanotechnology, Nanobiology, (1994), 3(1), 49–60.

(3) N.A. Baas, A Scheme for Synthesizing Complexity, In Chaos and Complexity, Eds. P. Bergé el al., Editions Frontiéres, Gif-sur-Yvette, 1995 (311–316).

(4) N.A. Baas, A Framework for Higher Order Cognition and Consciousness, In "Toward a Science of Consciousness", edited by S. Hameroff, A. Kaszniak and A. Scott, MIT-Press, 1996 (633–648).

(5) N.A. Baas, Thoughts on Higher Order Structures, ECHO-Amiens 1996, Yverts, Amiens 1996 (47–52).

(6) N.A. Baas, Categorical matter, Preprint, NTNU, 1996.

(7) N.A. Baas, Self-Organization and Higher Order Structures, In "From individual to collective dynamics",Gordon and Breach, 1997 (71 - 82).

(8) N.A. Baas, A.C. Ehresmann and J.-P. Vanbremeersch, Hyperstructures and Memory Evolutive Systems, International Journal of General Systems, vol 33(5), pp 553-568.

(9) N.A. Baas, B.I. Dundas and J. Rognes, Two-Vector Bundles and Forms of Elliptic Cohomology, (X-archives, math. AT/0306027, June 2003), in "Topology, Geometry and Quantum Field Theory, ed. U.L. Tillmann, London Math. Soc. LN 308, CUP, (18-45), 2004

(10) N.A. Baas and T. Helvik, Higher Order Cellular Automata, To appear in Advances in Complex Systems.

(11) J. Baez and J. Dolan, Higher-dimensional algebra and topological quantum field theory, J. Math. Phys, 36(11), 1995, 6073 - 6105.

(12) N. Bourbaki, Théorie des ensembles, Hermann, Paris, 1966.

(13) C. Ehresmann, Oevres complétes et commentées, Amiens, 1981.

(14) J. Gegenberg and G. Kunstatter, Using 3D Stringy Gravity to Understand the Thurston Conjecture, Preprint X-archives: hep-th/0306271, June 2003.

(15) A. Grothendieck, Pursuing stacks, manuscript, 1983.

(16) T. Leinster, Higher Operads, Higher Categories, book to appear at CUP in 2004.

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