

Figure 1

The system has several levels: an object A of level *n*+1 is the *binding* (or *colimit*) of at least one pattern³ Q of linked objects of levels $\leq n$, which it functionally represents. It means that there is a *collective link* (c_k) from Q to A (cf. Fig. 2) through which any collective link (g_k) from Q to C factors. Then A has ramifications (cf. Fig. 1) down to level 0, and its complexity order is the length of its shortest ramification (it is $\leq n+1$).

BINDING PROCESS. HIERARCHY If A and C bind Q and P, there are *n*-simple links

g from A to C which bind a cluster G of links between components of Q and P, correlated by the distinguished links of Q and P (cf. Fig. 2). A simple link just represents a cluster as an entity, thus translates properties of the lower components of A and C

from a subcategory of H_t to $H_{t'}$.

n-simple link =gCollective lin 0 cluster G 0 Figure 2



Figure 3

The system satisfies the degeneracy property if there are multiform objects C binding 2 patterns which are functionally, but not structurally, equivalent. Then there are n-complex links composing simple links binding non-adjacent clusters (cf. Fig. 3). They *emerge* at level *n*+1 depending on the whole structure of the lower levels.

acterizes the systems having some objects whose complexity order is > 1.

MULTI-SCALE DYNAMICS A MES has a multi-scale self-organization depending on the cooperative and/or competitive interactions between a net of specialized functional subsystems, the coregulators, having their own complexity level and their own discrete timescale extracted from the continuous timescale of the system. It has a central flexible memory Mem which develops over time and possibly contains a robust though flexible internal model AC of

the system (cf. Fig. 1). Links are labelled with a propagation delay and a strength (in \mathbf{R}_{\perp}). Each CR acts at its own rhythm as a hybrid system: 1. At a step of its discrete timescale, it forms its landscape (modeled by a category) with the partial information it can access, evaluates the result of the preceding step, and selects an adapted procedure with the help of Mem. 2. The realization of this procedure during the continuous time of the present step is directed by differential equations implicating the propagation delays and strengths of the links, and it should move the landscape to an attractor. These operations should respect the structural temporal constraints of the CR.

The procedures of the various CRs at a given time may not fit together. The correlation between their dynamics is ensured by an equilibration process, the interplay among the CRs, possibly by-passing the procedures of some CRs and causing dysfunction (temporary 'fracture' or dyschrony) to them. In particular there is a 'dialectics' between the dynamics of CRs with heterogeneous complexity levels and rhythms.



The repair of dyschrony backfiring between very different levels may lead to a change of the rhythm of some CRs. For instance, we have proposed a theory of aging for an organism through such a cascade of desynchronizations of CRs of higher and higher levels.

DEFINITIONS

1. A category is an oriented (multi-)graph with an internal (partial) composition which maps a path (f, g) from A to B on an edge fg from A to B, is associative and such that each object has an identity; a vertex is called an object, and an edge a morphism (or a 'link'). In particular a (multi-)graph is included in the category of its paths, and each category is the quotient of a category of paths. 2. A functor F from a category H to H' maps an object A of H to an object FA of H', a link from A to B on a link from FA to FB, and preserves the composition. 3. A pattern (or diagram) P in a category consists in a family of objects (P_i) and distinguished links between them.

DEGENERACY. COMPLEXITY. EMERGENCE

COMPLEXITY THEOREM. Degeneracy char-

In Memory Evolutive Systems: Hierarchy, Emergence, Cognition (Elsevier, 2007) and earlier papers we have proposed the model MES for evolutionary multi-scale autonomous systems, based on a 'dynamic' category theory, integrating multiple temporalities. The configuration of the system around time t is modeled by a hierarchical category¹ H, and the change from t to t' > t by a 'transition' functor²

MES: A theoretical approach to

multi-scale emergence and dynamics

Andrée Ehresmann* and Jean-Paul Vanbremeersch *Université de Picardie Jules Verne http://pagesperso-orange.fr/ehres http://pagesperso-orange.fr/vbm-ehr



In a MES the transitions will result from

complexification processes relative to pro-

cedures to 'suppress' some objects, 'absorb'

EMERGENCE THEOREM. The degeneracy

property extends to a complexification. In a

MES, it allows for the emergence over time

of an intertwined hierarchy of elements of

increasing complexity order, e.g. higher

cognitive processes in the model MENS of a

neuro-cognitive system.

external objects, 'bind' some patterns.