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HIERARCHICAL EVOLUTIVE SYSTEMS

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ABSTRACT

Memory Evolutive Systems represent a mathematical model for natural self-organizing systems, such as biological or sociological systems, which are open, have a hierarchical structure with components of increasing orders of complexity, and develop a memory enabling them to store and retrieve informations about their environment, and use their past experiences to adapt to it.

This model (introduced in preceding papers) is based on a recent domain of Mathematics, Category Theory, which provides an adequate setting for studying the dynamical interconnections between the different complexity levels and the formation of more and more complex units by integration of simpler ones.

INTRODUCTION

The study of neural networks has been very active these last years. However most models cannot explain the development of higher cognitive processes, and it seems new methods are needed for it. The Memory Evolutive Systems presented here are a tentative model to allow for computations on the "synchronous assemblies of neurons" which are thought to be the basis of higher neural functions (cf. [9]). It has been obtained by refinement of a general model for autonomous natural systems, called a Hierarchical Evolutive System, introduced in [3]. Based on a less classical mathematical domain, Category Theory, it relies on two interrelated categorical notions: the (inductive) "limit" operation (cf. [7]) to explain the formation of complex units by binding together of more elementary ones, and the completion of a category by the adjunction of certain types of limits, to describe the dynamical evolution of the system by successive complexifications.

In [4] we have considered the case in which the dynamics depends on the interaction of the system with an internal Center of Regulation (CR) which receives partial informations on the present state and on the external constraints. This center forms its own representation of the system, called its landscape, and uses it to direct a stepwise trial-and-error learning process.

The next step toward the modelization of a neural system has consisted in adding to the system a procedural memory in which past experiences are stored and may be retrieved later on. Moreover a natural system has not a unique CR, but all a hierarchy of competitive CR's. Whence the notion of a Memory Evolutive System (cf. [5]) that we develop here, in which the evolution is modulated by the interactions between a family of CR's. Each CR has a specific time scale and complexity order, and it interacts with the system through its own landscape. But the strategies of heterogeneous CR's may be conflictual, and there develops a "dialectics" between them which is the basis for the generation of more complex processes.

The general notions will be first introduced in the particular case of a neuronal system, so that their practical meaning may be more easily grasped.

1. THE CATEGORY OF NEURONS

A category consists of objects and arrows between them, forming an oriented graph; there is given a rule to combine two successive arrows in another one, so that a path of the graph gives the same arrow by combination whatever be its 2-2 decomposition, and each object has an identity arrow (cf. [7]). The state of a natural system at a given time t will be modeled by a category K_t which represents its internal organization: the objects

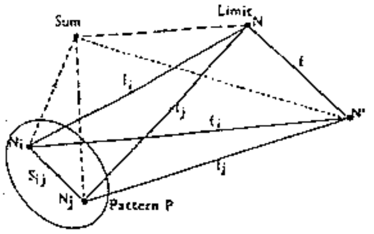

FIGURE 1

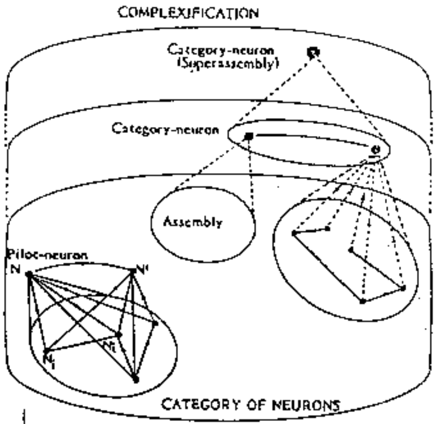

FIGURE 2

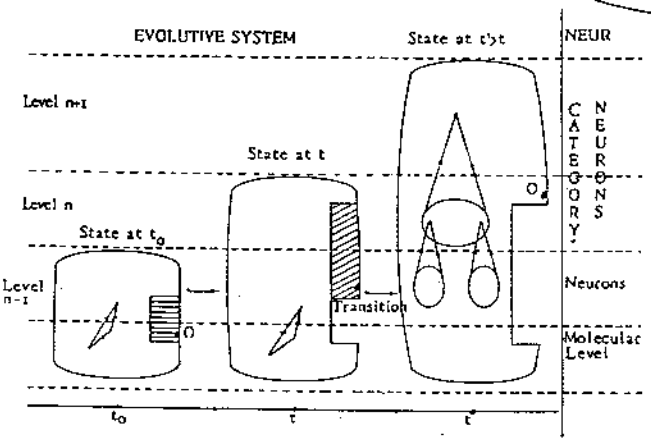

FIGURE 3

FIGURE 4

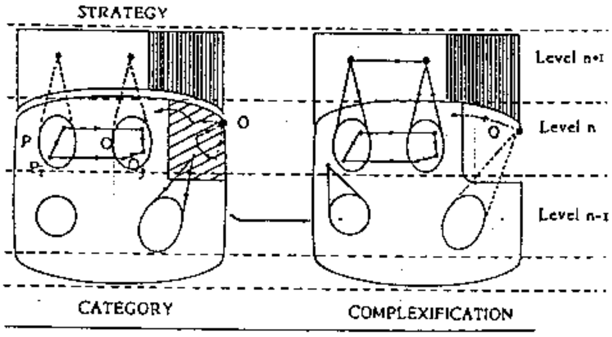

figure the components, the arrows (called links) their interrelations such as information or energy channels, constraints, causal relations. The combination ensures the transitivity of transfers.

In particular, to describe the state of a neural system at a given instant t, we define the *category of neurons* as follows: We start from the neurons and the synapses between them; a neuron is characterized by its activity (or instantaneous firing rate), a synapse by its strength. The objects of the category are the neurons, the links from N to N' are classes of synaptic paths having the same strength; two paths have the same strength if the activity of N generates the same effect on N' that it is propagated along one path or the other. Successive paths are combined by concatenation.

An external stimulus or an internal process activates a pattern of neurons N_i, linked by some specific synaptic paths. For simple enough patterns, there exists a single neuron N, called a *pilot-neuron* (or a cardinal cell by Barlow), which subsumes the pattern; it means that the N_i's are strongly linked to N, and if N is linked to another neuron N', it propagates the same activity to N' as the whole pattern when it is synchronically firing. We'll see (§2) that it means N is a limit of the pattern in the category of neurons (cf. Figure 1). Such feature-detector cells exist in the visual cortex, for instance some individual neurons in the area 17 detect segments of a given direction, or angles.

However complex enough patterns have no pilot-neuron (there are no "grandmother neurons" except for some particularly significant features). In this case, the storage and ulterior recognition of the stimulus will depend on the modification of its specific links which become stronger and so increase the cohesion of the pattern. Thus in time it becomes a synchronous assembly of interconnected neurons in Hebb's sense [6], and the stimulus is recognized by the coherent oscillation of this assembly, or, in case of noise or damage, of a modified but close enough neuronal group [9].

One of the interests of using Category Theory is that it is better equipped to handle this process than most classical methods. A natural idea would be to represent the assembly by a simple graph, but this representation would not have the required stability. In our model, we construct, by the complexification process (cf. § 2) a larger category, the category of neurons and category-neurons (Figure 2), in which a synchronous assembly which has no limit in the category of neurons acquires a limit; this limit may be thought of as a conceptual (but not physical) unit, called a *category-neuron*, and it is also the limit of the other close patterns that yield the same output. Hence it provides an accurate storage of the corresponding stimulus or process, yet allowing for a flexible, non strictly deterministic recognition process (thus satisfying the "degeneracy" property formulated in [2]).

The construction of the larger category helps also to define the correct notion for the links between two synchronous assemblies, so that computations on them are easily performed. For instance, it will be possible to compare two assemblies and to determine to which extent an assembly may be damaged without modifying its overall activity. And the complexification will be iterated to form superassemblies by the binding of less complex pre-existing ones, thus leading to the stable representation of more and more complex stimuli or processes.

These results depend on the following categorical notions.

2. COMPLEXIFICATION

A *pattern* in a category is a family of objects N_i with some specified links between them. A *collective link* from this pattern to an object N' consists of individual links from the N_i's to N', interconnected by the specific links; hence it models how an information or a constraint is globally transmitted to N' in a coherent way by all the components of the pattern.

In some cases (not always), the pattern looked at as an entity is "really" represented in the category by a unique object N. More precisely, an object N is a *limit* (or *cohesive binding*) of this pattern if there exists a canonical collective link from the pattern to N and if the links from N to N' are in 1-1 correspondance with the collective links from the pattern to N'. If there exists such an N, it may be considered as a *complex object* admitting the pattern for its internal organization.

If we "forget" the specified links and just keep the N_i's, then the limit of this reduced pattern would be the *sum* S of the N_i's (Figure 1). The coherence and the constraints introduced by the specified links is measured by the *comparison link* from S to N (cf. [3]). It explains the emergence of new properties for the complex object N not shared by its components, while the properties of the sum are just those of its components N_i's. It is important to notice that two patterns may have the same limit, that means their overall organization is similar.

A natural system changes in time. Its evolution is modeled by an *evolutive system*: it consists of a sequence of categories K_t representing its successive states, connected by transition functors from K_t to $K_{t'}$ which represent the change from the instant t to a later instant t'.

An evolutive system is a *hierarchical system* [3] if the objects of each state category are classified in complexity levels, so that an object at level n+1 is a limit of a pattern of level n objects (cf. Figure 3). Then it has also a more complicated internal organization of the lower levels. And each object presents a double face: elementary component of a higher level object, or complex entity representing the integrality of its internal organization.

The complexification process used in §1 is an adaptation of the completion of a category by some limits. It has been introduced in [3] to model the evolution of natural self-organized systems, which is mainly shaped by recurrence of the four "archetypal singularities" (cf. Thom [10]): birth, death, collision and scission. For example, for a cell, they correspond to absorption of external molecules, exocytosis, synthesis of new products, scissions of components.

To describe this situation, we start with a category on which there is given a *strategy* which requires that some patterns without a limit be "binded", that some objects or limits be suppressed and that some external elements be added. Then we explicitly construct a larger category, the *complexification*, in which the objectives of the strategy are fulfilled in the most economical way, both on a theoretical and an energy-cost basis (Figure 4). In particular each pattern to bind acquires a limit in this category, which may be thought of as the pattern itself taken in its integrality as a new object of a higher complexity level, this object being a stable representation not only of the particular pattern but of the whole class formed by the similarly organized patterns.

This construction is not a purely theoretical device. Its practical interest is to provide an explicit construction of the "good" links between the new objects, and hence between the patterns they are binding (e.g. between two synchronous assemblies of neurons if we complexify the category of neurons). And the construction can be iterated to get more and more complex objects by successive complexifications. This kind of operation, akin to the passage from a language to a higher order one, is at the base of any knowledge: new terms are defined from more primitive terms, and they become primitive terms for a higher complexification. It will be the basis of a stepwise trial and error learning process for evolutive systems, and in particular neural systems.

3. THE LEARNING PROCESS.

Now we are going to study how a neural system functions. The category of neurons and category-neurons changes in time: some neurons or synapses are damaged, new limit-neurons are recruted, and the cognition process consists in adding category-neurons representing synchronous assemblies, which form a hierarchy of conceptual units with higher and higher complexity levels. These modifications rely on molecular mechanisms which, translated at the neuron level, change the activity of the neurons and the strength of the synapses, according to the following rules (which lead to equations and to simulations we have no time to indicate here). The activity of a neuron at a given time is a bounded above function of the sum of the activities of the neurons linked to it,

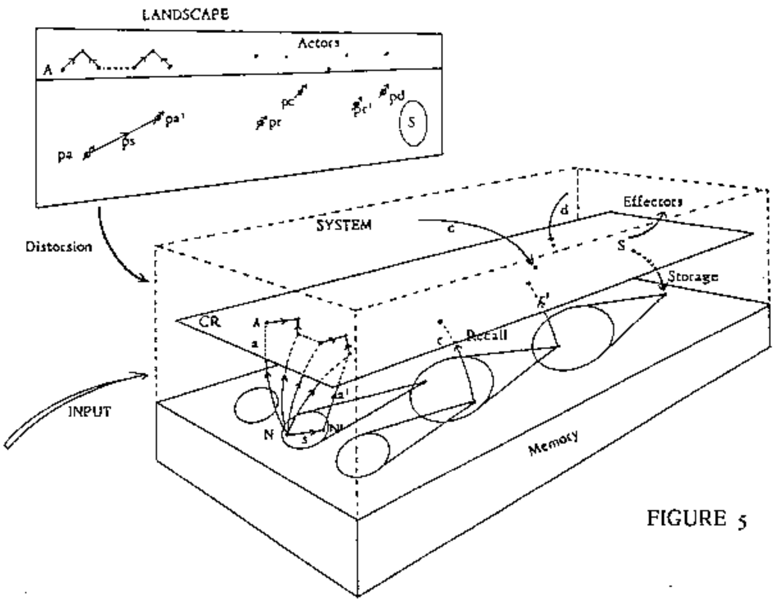

FIGURE 5

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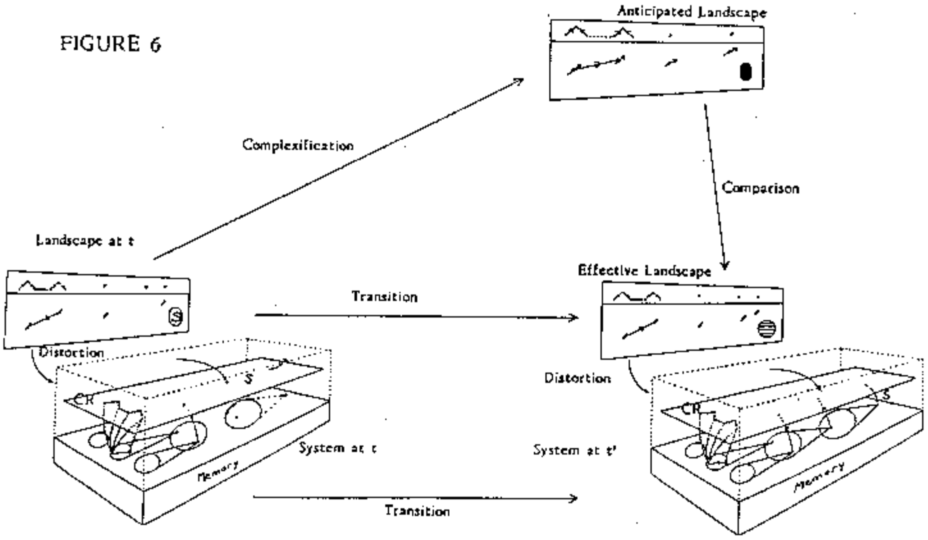


pondered by the strength of the links. The strength of a synapse is increased if the activities of its presynaptic and postsynaptic neurons are correlated, reduced if they are anti-correlated (that seems physiologically relevant, cf. [8]). Learning occurs by the differential amplification of some patterns which modifies the strength of the synapses and increases their threshold, with a local selection winner-takes-all rule ("neural Darwinism" [2]).

From a categorical point of view, we model the neural system by a hierarchical evolutive system with several levels, from the level of its molecular organization up to levels of category-neurons representing synchronous assemblies or super-assemblies. The dynamics of the system by auto-regulation and learning is described stepwise as follows (cf. [4,5]).

We distinguish two hierarchical sub-systems: a *Memory* and a *Center of Regulation* CR (corresponding to associative areas, e.g. basal ganglia), with simple neurons in the lower levels, and category-neurons representing synchronous assemblies or superassemblies of neurons in the higher ones. The center has both afferent and efferent links with the memory, and with effectors (motor neurons).

At a given time, the receptors (sensory cells) extract local features from the environment and the center receives informations on the state of the system thanks to links from the receptors, the effectors and the memory. These informations are modelled by a new category, called the CR *actual landscape* (cf. [4,5]): its objects are classes of links from the system to the CR which convey the "same" information, i.e. which are interconnected by links in the CR (Figure 5). The distortion between the landscape and the system is measured by a functor.

A change in the landscape at t will prompt the beginning of a new step. Then, taking into account informations on the internal and external constraints and recalling similar past experiences, a strategy is chosen on the landscape to activate some assemblies or superassemblies of neurons, to control the effectors in an adequate way and to facilitate the storage of the present context in the memory. This strategy consists in suppressing some elements and adding category-neurons to some patterns. The following anticipated landscape should be the corresponding complexification. But the effective one may be different, because the goals of the strategy are not necessarily fulfilled. Indeed, the strategy has been selected on the landscape and is only repercuted to the system with a distortion. We measure the difference between the anticipated and the effective landscapes at the CR level by the *comparison functor* (Figure 6), and describe how to feedback the results to the memory, so that they may be retrieved later on (cf. [4,5]).

In this section, we have focused on a particular CR. But in fact to explain the development of higher order cognitive processes we'll have to consider all a family of interacting CR's.

4. MEMORY EVOLUTIVE SYSTEMS

The dynamics of autonomous systems such as biological systems is shaped by the interactions between internal regulatory organs with different complexity levels and propagation delays. To model this situation, we use the following notion [5].

A *Memory Evolutive System* is an evolutive system equipped with a hierarchical evolutive sub-system, called the Memory, and a family of Centers of Regulation (CR) satisfying the following conditions (Figure 7). Each CR has a differential access to the system and in particular to the central memory through afferent and efferent links. Its objects, called *actors*, are of a specific complexity level and it functions with a given propagation delay, forming an evolutive system with its own time-scale the length of the steps being related to the propagation delay. The time-scales increase with the complexity level. Parallel CR's with the same complexity level may have different time-scales. Higher levels CR's are associative, in that they collect informations through patterns of lower level actors. For instance in a neural system, there are specialized visual or auditive CR's, but also associative CR's regrouping them, up to the conscious level.

Each CR directs a *trial and error learning process* (cf. [5]) which is descri-

MEMORY EVOLUTIVE SYSTEM

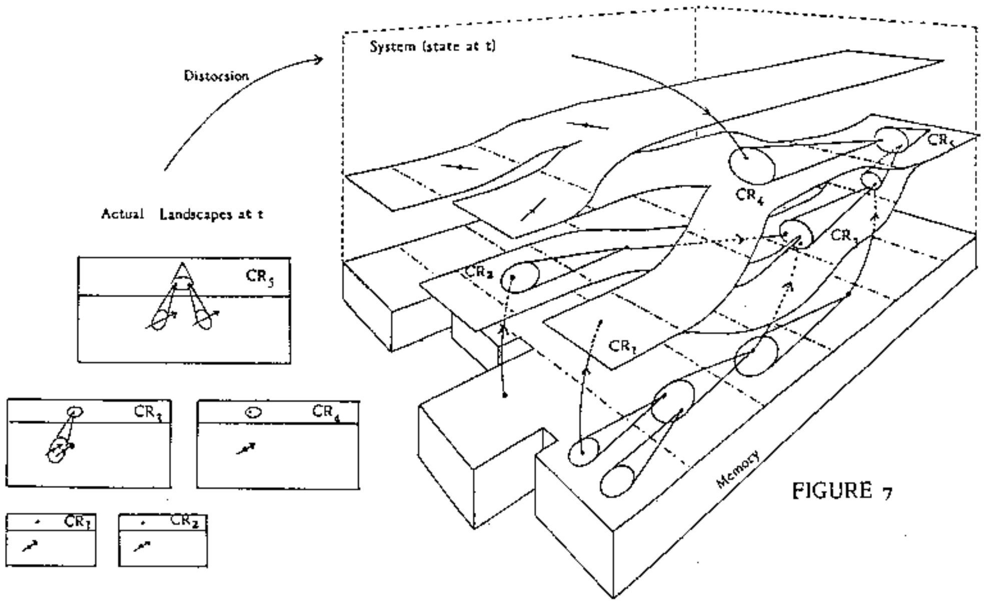
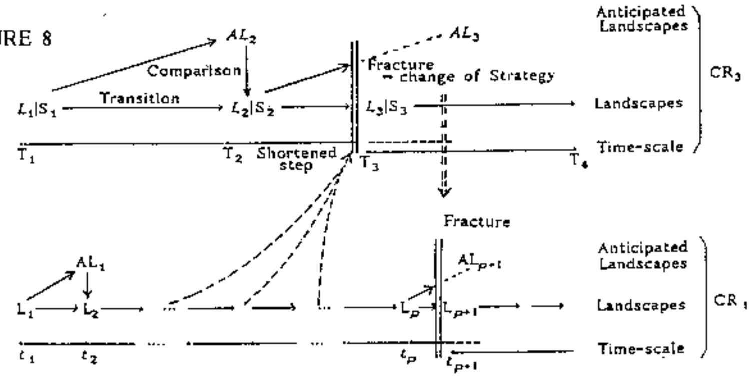


FIGURE 7

FIGURE 8



Accumulation of lower level changes causes a fracture to the higher level center of regulation CR_1. The subsequent center of regulation CR_3 change of strategy backfires on the CR_1 landscape bed as above for the neural system, and in which it has a triple function: as an observational system, it forms its *actual landscape*, which gives a particular internal representation of the system. As a decision and control organ, it selects a strategy on this landscape to command effectors and, at the end of the step, to memorize the strategy and its results, evaluated thanks to the *comparison functor* (Figure 6). In time, these memorized strategies constitute a growing *procedural memory*, in which they can be compared and eventually weighted.

At a given time, the strategies devised by the different CR's on their respective landscapes are only repercuted to the system with a distortion and they become competitive, and eventually conflictual. In that case, some CR's will not be able to enforce their current strategy and they'll have to change it before the normal completion of the present step; we say there is a *fracture* in their landscape (analytically it would be expressed by a catastrophe in Thom's sense). Several types of fractures are investigated (cf. [4,5]). For instance, during one step of an associative CR, say CR3, a lower level CR goes through a sequence of shorter steps (Figure 8); the modifications generated by each of these steps are not directly perceived at the CR3 level, but their accumulation may disrupt the processing of the strategy devised by CR3, thus causing a fracture for CR3 and shortening its step. Conversely, a higher level fracture may backfire later on and force the lower levels to adopt a specific strategy. Other fractures come from a severe modification of the environment, or because of a conflictual standstill between two parallel CR's, which has to be deblocked by a higher level. So the evolution of the system is shaped by the *dialectics* between CR's which are heterogeneous by their complexity level and their time-scale, the bottom-top regulation being predominent on a short term, but modulated by up-down stringent controls that shape the evolution on the long run (cf. [5]).

In the case of a neural system, this model can explain the formation of *higher cognitive structures* through the interaction with the environment and the dialectics between heterogeneous CR's. Following Changeux [1] and Edelman [2], it might be conjectured that *consciousness* is an emergent property of the dialectics between a higher level CR and lower CR's which direct automatic processes. In particular, man develops a *semantic memory* formed by more and more complex category-neurons representing abstract concepts, controled by the higher CR associated to the language (Broca and Wernicke areas). Thought would be produced by sequences of fractures caused in this CR by lower ones, which impose a revision of the situation (through inter-levels loops [2]), yet leaving some indeterminism in the choice of the new strategy thanks to the flexibility of the category-neurons.

On a less conjectural side, the model helps understand some experimental results of Cognitive Science and suggests new questions to explore. It may also be adapted to study the dynamics of various biological or sociological systems, to design knowledge-based machines in Artificial Intelligence, or to describe knowledge acquisition in Epistemology.

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