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MULTIPLICITY PRINCIPLE AND EMERGENCE IN MEMORY EVOLUTIVE SYSTEMS

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Memory Evolutive Systems, introduced in preceding papers [1, 2], propose a mathematical model, based on Category Theory, for self-organized hierarchical natural systems, such as biological, neural or social systems. Their dynamics is partially regulated by the coordinated and/or competitive interactions between a net of internal Centers of Regulation (CR). Each CR operates at a specific complexity level and with its own time-scale. There is an interplay among the strategies repercuted to the system by the various CRs, and it may cause fractures to some particular CRs to ensure global stability.

Here we characterize one of the properties responsible for the generation of complex processes, namely the existence of 'multifold' objects admitting several non-equivalent decompositions into patterns of objects of a lower level. We show that this 'Multiplicity Principle' affords a greater plasticity to the dialectics between heterogeneous CRs, and that it is at the root of the emergence of objects and links of increasing orders. In neural systems it might explain how mental states supervene on physical states by a gradual unfolding.

Keywords: Category theory; memory evolutive systems

1. INTRODUCTION

Biological or social systems have many characteristics that differentiate them from material systems, such as their hierarchical organization with interactions between components of different scales, their dynamics by assembly and disassembly directed by overlapping internal regulations at various complexity levels and time-scales, their plasticity, eventually leading to the emergence of higher order processes. However most mathematical models developed for them inherit their methods from Theoretical Physics, mostly based on dynamical systems, chaos theory, thermodynamics or information theory. These models may give interesting results and simulations for a specific process in a welldelimited environment, when observables can be fixed once for all (Kampis [3]). But many authors (e.g. Rosen [4]) have emphasized that they are insufficient to deal with the more global evolution of complex systems.

So new mathematical tools seem necessary, and such a tool could be Category Theory. The Memory Evolutive Systems (MES), developed in a series of papers (e.g., [1, 2]) since 1986, constitute a model for complex systems which possesses the above characteristics. The state of the system at a given date is represented by a category, and the change of states by the process of 'complexification of a category with respect to a strategy', which describes the formation of 'complex' objects by assembly of patterns of pre-existing linked objects, and determines their interactions.

In this paper, we prove that these interactions can be of two kinds: the 'simple links' which just bind clusters of pre-existing links, and the more interesting 'complex links' which represent really emergent properties. The existence of complex links relies on the existence of 'multifold objects', which have several non-equivalent decompositions into patterns of linked objects (Multiplicity Principle).

We study the consequences of this principle on the dynamics of a MES, which is partially controlled by the interactions (cooperation and/or competition) between a net of internal control organs, the Centers of Regulation (CR). Each CR operates a stepwise process at its specific complexity level and with its own time-scale, but there is an interplay among the strategies that the various CRs try to enforce, and it may cause fractures for some CRs if their specific structural and temporal constraints cannot be respected. These constraints come from the fact that material transmission between heterogeneous CRs requires some delay and cost in energy (this aspect of the problem is well emphasized by Matsuno [5]).

The existence of multifold objects adds more plasticity to the 'dialectics' between CRs, and we prove that it may lead to the emergence of higher order objects through a non-reducible several steps construction, thus offering a model for an 'emergentist reductionism'. Applied to neural systems, this process explains how higher order cognitive processes may be formed, and it suggests an original approach to the brain/mind problem.

MES can be applied in different domains. In Biology, they give a model for the growth of a cell (which has been compared in [6] with the C8 theory of Chandler), or the development of an organism, or the speciation process. They seem in agreement with the Protobiology developed by Matsuno [5] as a Physics for Biology. In Sociology, they emphasize the role of structural and temporal constraints in the functioning of enterprises or societies (EV [7]). In Epistemology, they help describe the development of a theory and are used as a frame for a theory of interdisciplinarity (Lunca [8]).

2. EVOLUTIVE SYSTEM [1]

2.1. Categories and Functors

Our model is based on Category Theory, a recent domain of Mathematics introduced by Eilenberg & Mac Lane in 1945 to study the interrelations between different mathematical structures (e.g. Topology and Algebra, or Logic and Geometry). We just recall the following definitions and send back to usual books (e.g., Mac Lane [9]) for more details.

A *category* is an oriented graph, with possibly loops and several arrows between two vertices, on which there is given a 'composition law' associating to 2 successive arrows (f, g) another arrow $f \cdot g$ such that: 1. Associativity: each path of the graph has a unique composite whatever the way it is decomposed into sequences of 2 successive arrows. 2. Identity: there is a closed arrow 'identity of N' for each vertex N, the composite of which with any arrow with source or target N is this arrow. The vertices of the graph are called the *objects* of the category, its arrows the *morphisms* or (here) the *links* of the category.

A *functor* from a category to another one is a graph homomorphism compatible with the composition of links.

2.2. Evolutive Systems

To model a natural system, we define the notion of an Evolutive System, in which the state of the system at a given date t is represented by a category, and the change of states by a functor.

DEFINITION An *Evolutive System* (abbreviated in ES) is defined by the following data: a set of positive real numbers t or *reference time-scale*; for each t a category $\mathbf{K}_t$ called the *state-category at* t, and for each couple of dates (t,t') with $t<t'$ a functor *transition* from $\mathbf{K}_t$ to $\mathbf{K}_{t'}$ so that there is transitivity for these transitions. We suppose that each category $\mathbf{K}_t$ has an initial object 0 which models the objects lost before t.

In that state-category at t, the objects represent the different components of the system at this date, and the links the relations between them, which can be transfers of information or energy, topological or causal connections, constraints... The composition of links determines classes of paths which are functionally equivalent. For an object N, we consider that the links with source N represent its 'actions' on other objects or the messages it sends; while the links toward N model the messages it receives, or constraints imposed on it by other objects.

In natural systems, the transmission of informations or commands requires some energy and delay. We model this by the data of observables which associate to each link f of a state-category a real number called its *weight*, so that the weight of a composite be the sum of the weights of its factors. In particular we consider such an observable $p(f)$ which represents the *propagation delay* of f.

The categories modeling natural systems are often constructed from a labelled graph in which paths having the same summed labels are identified. For instance, the ES representing a neural system is constructed from the *category of neurons* in which the objects N represent the neurons, and the links from N to N′ represent the synaptic paths between them, two synaptic paths being identified if they transmit an excitation from N to N′ with the same force and same delay. (This category is implicit in Zeeman [10].)

2.3. Patterns and their Collective Links

The components of a natural system are not all equivalent; some can be considered as more 'complex' than others in the sense that they represent the concatenation of a pattern of 'more elementary' objects acting as a coherent assembly.

For instance a protein is more complex than its atoms, because it consists in an assemblage of interacting atoms arranged in a specific spatial conformation. The internal organisation of the protein will be modeled by a pattern in the category formed by atoms and molecules, in which the links represent chemical relations (covalence bounds, van der Waal forces,...), and the protein as such will be represented by the colimit (or binding) of this pattern.

DEFINITION A *pattern* (or *diagram*) in a category **K** is a homomorphism **P** from a graph I to **K**. The (indexed) images $\mathbf{P}_i$ of the vertices i of I are called the *objects* of the pattern, and the images of the arrows of I are their *distinguished links*. A *collective link* from **P** to an object N′ (or cone of base **P** and vertex N′) is a family (f_i) of links from each object $\mathbf{P}_i$ of the pattern to N′ such that, for each distinguished link x from $\mathbf{P}_i$ to $\mathbf{P}_j$, we have $x.f_j = f_i$. (Fig. 1)

A collective link models an action which requires that all the objects of the pattern cooperate through their distinguished links. Though a pattern is not an object of the category, it acts as an entity through its collective links. In some cases, this entity is internally reflected in an object of the category, called the colimit (or inductive limit [9]) of the pattern. For instance, a group of people with similar interest may coordinate their actions to realize some specific projects which they could not achieve if they acted separately. Their group can remain informal, or be institutionalized by an association with a legal statute. Such an association will be modeled by the colimit of the pattern they form in the category of individuals and social groups.

2.4. Colimit of a Pattern

DEFINITION The *colimit* (or *binding*) of a pattern is an object N, often denoted by colim**P**, such that there exists a *canonical* collective link (l_i) from the pattern to N and that each collective link (f_i) from the pattern to an object N′ is binded into a unique link f from N to N′ satisfying $f_i = l_i \cdot f$ for each object $\mathbf{P}_i$ of the pattern. (Fig. 1)

The colimit models the integration of the pattern into a single unity N. From upside-down, N may be thought of as a complex (or 'higher order') object which admits its own internal organisation of interacting components, represented by the pattern.

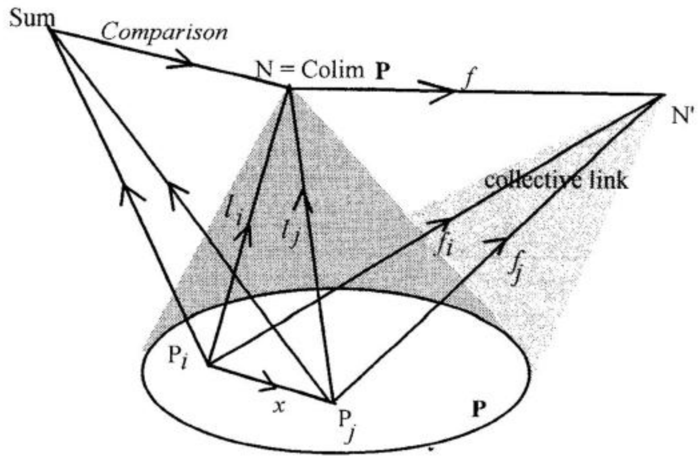

FIGURE 1 Collective link and colimit.

PROPOSITION *A pattern in a category has at most one colimit (up to an isomorphism). Two different patterns may have the same colimit* N; *a pattern having* N *for its colimit will be called a decomposition of* N. *In particular, a sub-pattern of a pattern may have the same colimit as the pattern itself.*

For example, an aminoacid is the colimit of each one of its synonymous codons in the category of molecules; or an ambiguous figure is simultaneously the colimit of the two patterns into which it can be decomposed. The proposition means that the colimit operation is many-to-one: an object N may have several different decompositions into patterns of linked objects. Roughly, the colimit 'forgets' the details of the organisation of the pattern and only retains which collective actions it can achieve, and these can be shared by other more or less different patterns.

2.5. Comparison with the Sum

The constraints imposed to the objects of the pattern by their distinguished links, but also the strength of their cooperation, can be evaluated by comparing the colimit to the simple amalgam, or *sum*, of the objects $\mathbf{P}_i$, which would be the colimit of the pattern obtained when their distinguished links in **P** are omitted. The sum classifies the individual actions of the objects $\mathbf{P}_i$, while the colimit classifies their collective actions made possible by their distinguished links.

PROPOSITION *There exists a link 'comparison' from the sum to the colimit, which factorizes the new properties emerging from the binding of the pattern. Its weight measures the gain in efficiency due to the formation of the colimit.* (Fig. 1)

For instance, the tetramer hemoglobin can be represented by the colimit of a pattern specifying its spatial conformation; the link comparison then measures the difference between the oxygenation rate of its 4 separate units and the oxygen fixation rate of the tetramer (cf. Di Cera [11]).

3. SIMPLE AND COMPLEX LINKS

3.1. Clusters and Simple Links

The interactions between the objects of two patterns **P** and **P**′ which are compatible with their distinguished links are modeled by the notion of a cluster.

DEFINITION *A cluster* from **P** to **P**′ is a maximal set of links between the objects of these patterns satisfying the following conditions:

1. For each object $\mathbf{P}_i$ of **P**, there exists at least one link from this object to an object of **P**′; and if there exist several such links, they are correlated by a zig-zag of distinguished links of **P**′, as indicated in Figure 2.
2. If a link belongs to the cluster, the links obtained by combining this link with a distinguished link of **P** or with a distinguished link of **P**′ also belong to the cluster.

Roughly, a cluster is a family of links from objects of **P** to objects of **P**′, well correlated by the distinguished links of the patterns, and such that each object of **P** transmits compatible informations or constraints to **P**′. It follows that:

PROPOSITION *If* **P** *and* **P**′ *admits colimits, a cluster from* **P** *to* **P**′ *is binded together into a link from* colim**P** *to* colim **P**′, *which will be called a* (**P**,**P**′) *–simple link binding the cluster.*

Indeed, it is easily proved that all the links of the cluster with source $\mathbf{P}_i$ have the same composite p_i when combined with the canonical links to colim$\mathbf{P}'$, and that all such p_i form a collective link to colim$\mathbf{P}'$ (cf. Fig. 2). This collective link is binded together into a link from colim$\mathbf{P}$ to colim$\mathbf{P}'$ (by the universal property of a colimit).

A simple link just 'institutionalises' the cluster, without adding any information not accessible at the level of the patterns. In Embryology, the induction of a population by another corresponds to the formation of a simple link.

3.2. Composition of Simple Links

PROPOSITION *The link ff' which is the composite of a* $(\mathbf{P}, \mathbf{P}')$ – *simple link f and a* $(\mathbf{P}', \mathbf{P}'')$ – *simple link f' is a* $(\mathbf{P}, \mathbf{P}'')$ – *simple link.*

Indeed, the composites of the links of the two adjacent clusters that f and f' bind generate a cluster from $\mathbf{P}$ to $\mathbf{P}''$, amd $f \cdot f'$ is the $(\mathbf{P}', \mathbf{P}'')$ – simple link binding this composite cluster. (Fig. 3)

PROPOSITION *If* $\mathbf{P}$ *has a colimit* N *and if there is a cluster from* $\mathbf{P}$ *to a sub-pattern* $\mathbf{Q}$ *of* $\mathbf{P}$, *then* N *is also the colimit of* $\mathbf{Q}$, *and* $\mathbf{Q}$ *is called a representative sub-pattern of* $\mathbf{P}$. *It follows that two patterns* $\mathbf{P}$ *and* $\mathbf{P}'$ *have the same colimit if there exists a zig-zag of clusters between them, each cluster connecting a pattern to a representative sub-pattern.*

The existence of a cluster from $\mathbf{P}$ to a sub-pattern $\mathbf{Q}$ means that each object $\mathbf{P}_i$ of $\mathbf{P}$ is linked to at least one object of $\mathbf{Q}$, and if it is

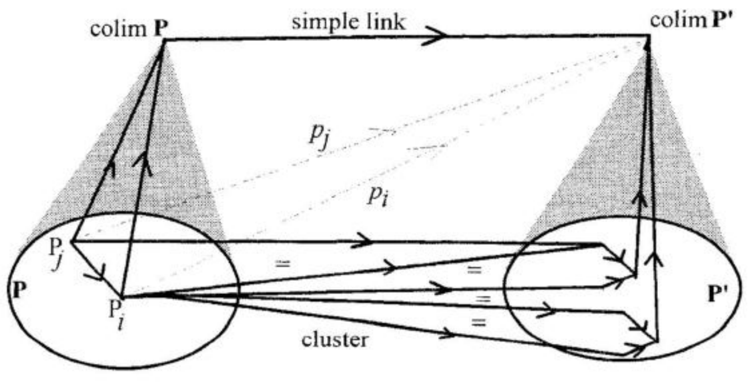

FIGURE 2 Cluster and simple link.

linked to several such objects, all these links transmit the 'same' information to **Q**. The typical example is given by the representatives of a nation: each elector votes for a list of representatives who agree on the policy to be conducted.

3.3. Non-equivalent Decompositions

A simple link binds a cluster between particular decompositions of the two objects N = colim**P** and N′ = colim**P**′. But we know that an object may have several decompositions; does a simple link remain 'simple' if we change the decompositions? The answer is negative: a (**P**, **P**′) – simple link is not always (**Q**, **Q**′) – simple for other decompositions **Q** of N and **Q**′ of N′. In particular, the identity of N is a (**P**, **P**) – simple link, but it is not always (**P**, **Q**) – simple. This result will have important consequences.

DEFINITION Two decompositions **P** and **Q** of an object N are said *equivalent* if there exists a cluster between them binding into the identity of N. If there is no such cluster (in one way or the other), we say that they are *non-equivalent*. An object admitting non-equivalent decompositions is called a *multifold object*.

For a multifold object, the 'switch' from one decomposition to a non-equivalent one measures the extent of fluctuation that its internal organisation may tolerate while its overall functions remain the same. An example of such a switch is the passage between the non-equival-

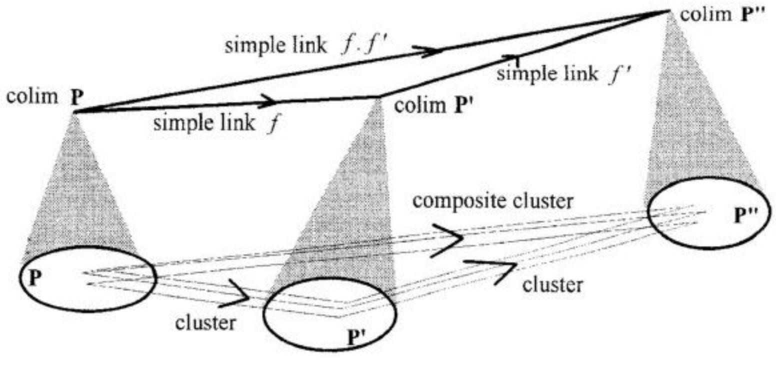

FIGURE 3 Simple composite of simple links.

ent genotypes of a species (i.e., genotypes with different alleles leading to the same phenotype).

3.4. Complex Links

We have seen that there is no difficulty to compose simple links binding adjacent clusters. The situation is different if we consider a (**P**, **P**′) – simple link from N to a multifold object N′, and a (**Q**′, **Q**″) – simple link from the same N′ to N″, with **P**′ and **Q**′ being non-equivalent decompositions of N′. These 2 links must have a composite from N to N″ (by definition of a category), but this composite may not be 'simple' in the sense of binding together a cluster (Fig. 4). Such a link will be called a *complex link*. For instance, the communications between authors and subscribers of a Journal form a complex link, mediated by the complex switch between Editors and Publishers. More generally, we define:

DEFINITION A *Complex link* from N to N″ is the composite of a sequence of 'simple links' binding nonadjacent clusters, and which does not bind a cluster between decompositions of N and N″ .

The simple links correspond to properties of the objects they connect for which all the information comes from the cluster they bind. Complex links connect the objects, not 'directly', but by the media of multifold objects, each of them intervening with two non-equivalent

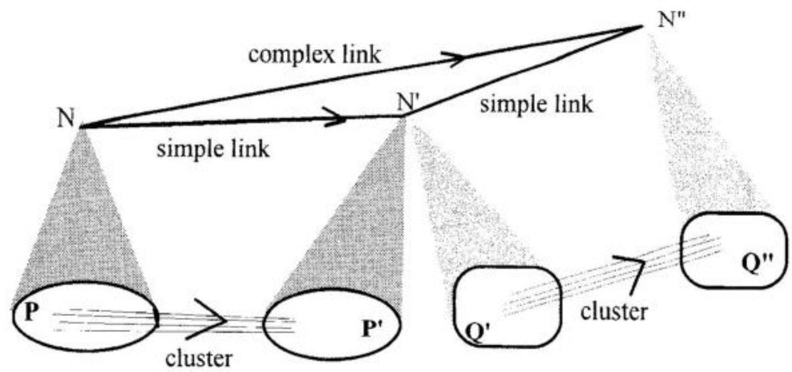

FIGURE 4 Complex link.

decompositions, and the switch between these decompositions introduces more plasticity and emergent properties with respect to the clusters. Generally, a composite of complex links is a complex link.

A concrete example of a complex link in the category of individuals and social groups is the composite of the (**P**,**P**′)-simple link 'offer' from a group of farmers colim**P** to (the purchasing service colim**P**′ of) a cooperative, with the (**Q**′,**Q**″)-simple link 'selling' from the (selling service colim**Q**′ of the) cooperative to a group colim**Q**″ of consumers. This link models the indirect operation of selling from farmers to consumers mediated by (the two different services of) the cooperative. For a mathematical example, topological spaces can be represented as the geometrical realizations of different simplicial complexes; then simple links correspond to simplicial maps, complex links to continuous maps.

4. ITERATED COLIMITS AND THEIR (NON-)-REDUCTION

An object A may have for decomposition a pattern **R** in which each object A_i admits its own decomposition into 'more elementary' objects. What is the relation between A and the components of the A_i? In particular, in which case is it possible to translate the correlations imposed to the A_i by the 'horizontal' links which **R** distinguishes into constraints imposed on their components, so that A becomes 'reducible' to the colimit of an appropriate pattern linking these simpler objects?

4.1. Iterated Colimits and Ramifications

DEFINITION If A is the colimit of a pattern **R** of linked objects $\mathbf{A}_i$ and if each $\mathbf{A}_i$ is the colimit of a pattern $\mathbf{P}_i$, we say that **A** is a *2-iterated colimit of* ($\mathbf{R}$,($\mathbf{P}_i$)), and that ($\mathbf{R}$,($\mathbf{P}_i$)) is a *ramification* of **A** of length 2, or 2-ramification. (Fig. 5)

The ramification represents a 2-stages internal organisation which determines completely the links from A to other objects in two steps. Looking 'upside-down', the multiplicity of decompositions of an object A implies that A may have several, possibly non-equivalent,

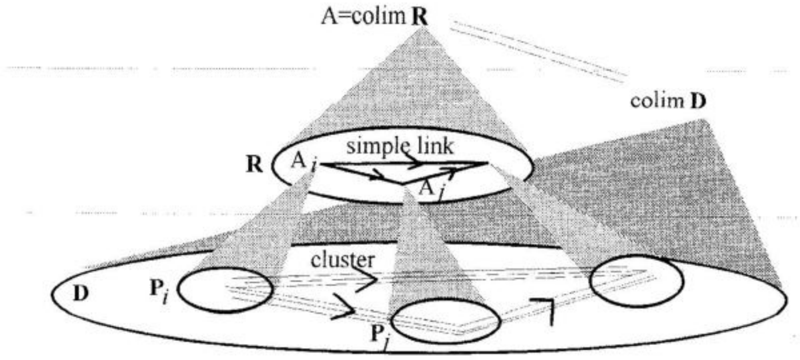

FIGURE 5 Reducible iterated colimit.

2-ramifications, and their number depends on the numbers of the decompositions of A and of each of its components. More generally

DEFINITION *A k-iterated colimit* is defined recursively: it is the colimit of a pattern of which each object is itself a $(k-1)$-iterated colimit. A *k-ramification* of an object A is the data of a decomposition of A and of a $k-1$-ramification of each component of this decomposition.

A ramification equips A of a kind of fractal strucutre, in which the components of each intermediate stage are themselves ramified, but moreover with correlations between these ramifications coming from the constraints introduced by the 'horizontal' links distinguished between these components.

Each decomposition of A attributes particular values to some characteristic features of A, so that its components play the rôle of the slots of a frame in the sense of Minsky [12]. A k-ramification gives more possibilities to fill the slots, each slot allowing various choices, and the same process being repeated at each of the k downward stages. In other terms, the number of stages increases the number of liberty degrees, with possible multiple switches at each stage. For instance, A may represent the menu in the restaurant scenario; this menu is first described by its general composition, say entree, meat, fromage and dessert, in this order; or (another decomposition) soup, fish, fruits. But then we may refine the choices, e.g. vegetables or ham as an entree, and anew mushrooms or tomatoes as vegetables, and so on.

4.2. Reduction of an Iterated Colimit

First we consider an object A in a category, with a ramification (**R**, (**P**$_i$)) of length 2. Thus the actions of A correspond to the collective actions of the objects of **R** cooperating through their distinguished links in **R**. Moreover each of these objects is by itself the colimit of a pattern **P**$_i$ so that it acts through the collective links of this pattern. Now we ask if it is possible to 'skip off' the intermediate stage represented by **R**, so that A becomes the (simple) colimit of a 'large' pattern containing the different patterns **P**$_i$. The following theorem shows that the answer to this 'reduction' problem relies on the distinction between simple and complex links.

THEOREM *Let* (**R**, (**P**$_i$)) *be a 2-ramification of an object* A. *If all the links of* **R** *are simple links binding clusters between the patterns* **P**$_i$, *then* A *is also the colimit of a pattern* **D** *in which the objects are all the objects of the different* **P**$_i$. *Conversely, if there exist complex links in* **R**, A *may have no decomposition into such a pattern.*

The proof relies of an analysis of the two cases:

1. (Fig. 5) Let us suppose that all the distinguished links of the pattern **R** are simple links binding clusters between the patterns **P**$_i$. Then we construct a 'large' pattern **D** in which the objects are all the objects of the different **P**$_i$, and the links come from the distinguished links of the patterns **P**$_i$ and from the links of all the clusters between them which are binded by the links of **R**. It is proved that this pattern **D** has the same colimit A as **R**. So, in this case, the object A which was initially described as an iterated colimit (hence requiring a 2-steps construction) can also be described directly (in only one step) as the colimit of **D**.
2. (Fig. 6) The situation is different if some of the distinguished links of the pattern **R** are complex. In this case, if we decompose the complex links into composites of simple links (belonging or not to **R**), and if we consider as above all the clusters associated to these simple links and all the patterns connected by these clusters, we still obtain a large pattern **D**; but A need not be its colimit. In other terms, in this case, A is 'essentially' an iterated colimit with respect to the objects of the patterns **P**$_i$, and there is no possibility to 'lower' the constraints imposed by **R**.

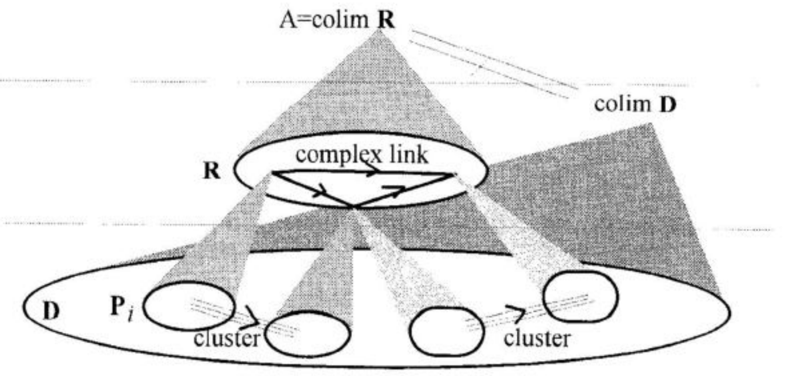

FIGURE 6 Non-reducible iterated colimit.

5. HIERACHICAL SYSTEM. HIGHER ORDER OBJECTS

5.1. Hierarchical Systems [1]

Iterated colimits come from the consideration of natural systems which have a hierarchy of more and more complex components, so that the components of a level have an internal organisation into components of lower levels. For instance, in the ES modeling an organism, we distinguish successively the atomic, molecular; sub-cellular, tissues and organs levels.

DEFINITION *A hierarchical system* (HS) is an ES in which the objects are partitioned into a finite number of levels, say from 1 to m, so that an object of the level $n+1$ is the colimit of at least one pattern of linked objects of level n. (Fig. 7)

A link between objects of level n is said to be of level n. There are also links from objects of lower to higher levels and vice-versa, so that the different levels are 'intertwined'. Among the links, we distinguish: (i) the *simple* links, which are the links of level 1 and the links binding a cluster between patterns of lower levels, and (ii) the *complex* links which are not simple though being a composite of simple links binding non-adjacent clusters.

An object of level n has a double rôle ('Janus'): it is a 'complex' object by comparison with its decompositions into patterns of level

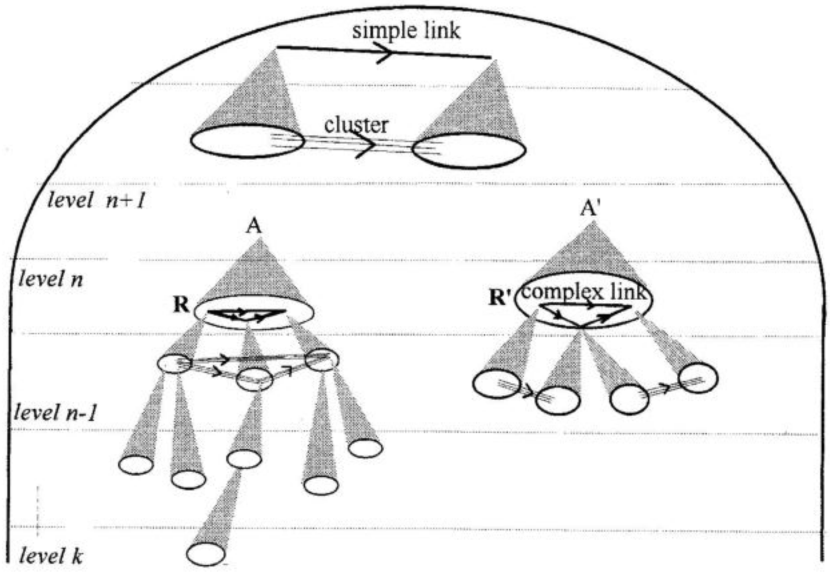

FIGURE 7 Hierarchical system.

$n-1$; but it is a 'simple' object when it is considered as the component of a pattern of level $n+1$ (whence the ambiguity of the word 'complex'!)

5.2. Reducible and Non-Reducible Objects

Let A be an object of level $n+1$, for $n>1$. It is the colimit of at least one pattern $\mathbf{R}$ of linked objects of level n. But each of the objects of $\mathbf{R}$ is itself the colimit of a pattern $\mathbf{P}_i$ of linked objects of the level $n-1$, so that A is the iterated colimit of ($\mathbf{R}$, ($\mathbf{P}_i$)). Proceeding down the ladder, for each $k<n+1$, A is represented as the $(n+1-k)$-iterated colimit of patterns of level k via all a sequence of intermediate level patterns, i.e., it admits $(n+1-k)$-ramifications with ultimate components of level κ. We are going to study in which case the length of such a ramification can be 'reduced'.

DEFINITION An object A of level $n+1$ is said to be *k-reducible* (respectively *k-multi-reducible*), for a $k<n+1$, if it can be represented

as the colimit of at least one pattern (respectively of several non-equivalent patterns) of level k.

An object A of level $n+1$ is always n-reducible, since it is the colimit of at least one pattern of level n, but it is n-multi-reducible only if it is a multifold object admitting non-equivalent decompositions of level n. It is k-reducible if, among all its ramifications down to the level k, there is one which 'skips all the intermediate stages', i.e., if A can be directly represented as the colimit (and not only as an iterated colimit) of a 'large' pattern **D** of level k coding for all the informations explicited by intermediate levels.

THEOREM *If all the links of the HS are simple, each object is k-reducible, for each k less than its level. But if there exist multifold objects, there may exist objects of level* $n+1$ *which are not k-reducible for some* $k<n$.

We can suppose $k=1$ (by 'forgetting' the lower levels), and prove the first assertion by iteration. Each object of level 2 is the colimit of a pattern of level 1, hence is 1-reducible. Let us suppose that each object of level k is 1-reducible and that A is an object of level $k+1$, by definition, it is the colimit of at least one pattern **R** of level k and the recurrence hypothesis entails that each object of **R** is itself the colimit of a pattern of level 1. If all the distinguished links of the pattern **R** are simple, the theorem of Section 3 asserts that A is also the colimit of a 'large' pattern **D** of level 1; hence A is 1-reducible. The same theorem shows that A may be non-reducible if there exist complex links in **R**.

5.3. The Problem of Reductionism

This problem asks if the study a higher level object can be reduced to that of its components of lower levels. For instance, is molecular Biology sufficient to explain the global organism? The preceding results allow to give some answer.

Let A be an object of level $n+1$. If it is k-reducible so that it is the colimit of a pattern of level k, its properties are entirely determined by the collective links of this pattern, in an 'algorithmic' (or linear) manner. If A is not k-reducible, its properties can still be deduced from those of its components of level k, but only progressively, through a progressive unfolding along the different stages of a ramification in which (some of) the 'horizontal' links are complex; so intermediate multifold objects intervene, with a possibility of switches between their non-equivalent decompositions which has a 'non-linear' bearing on the comportment of A. In this case, the deduction of the properties of A from those of its components of level k cannot be said to be algorithmic, even if it is well described in a 'geometrical' (or 'morphological') way. In any case, there is a reduction to the level k, but with complex properties emerging at each intermediate level. We could speak of an *emergentist reductionism* (in the sense of Bunge [13]), while a pure reductionism would require that there are no multifold objects.

5.4. The Multiplicity Principle

The preceding analysis shows that the existence of multifold objects characterizes the HS with non-reducible higher order properties, whence we introduce a

DEFINITION We say that the HS satisfies the *Multiplicity Principle* (degeneracy in the sense of Edelman [14]) if: 1. There exist n-multi-reducible objects. 2. An object may belong to several patterns of level n which have different colimits of level $n+1$.

By analogy with the definition of entropy as the logarithm of the number of microstates of a gaz in Thermodynamics, we define the *k-entropy* of an object A as the number of classes of its non-equivalent ramifications down to the level k. This entropy measures the flexibility of A, in the sense of the number of its different internal organisations. But (not agreeing with Rosen [15]), we do not consider that it measures the 'real' complexity of A. The k-complexity of A would be measured by its k-order defined as follows.

DEFINITION If p is the smallest length of a ramification of A down to the level k, we say that the *k-order* of A is equal to $p+1$.

This order determines the number of intermediate stages necessary to reconstruct A from the level k on (a similar question is studied by Klir [16] in his reconstructibility theory). From the above Theorem, it follows that, if the HS satisfies the Multiplicity Principle, it may exist objects of level $n+1$ whose 1-order is $n+1$. In particular, there may exist objects of level 3 which are of 1-order equal to 3. This answers the question raised by Baas [17] of the existence of such 'hyperstructures' of order 3 (in his terminology).

5.5. Some Concrete Examples

Let us give some concrete examples which show the difference between (1-) reducible and non-reducible objects. In the HS modeling the occidental society, with the level of individuals and different levels of social groups, an "Europe of people" would be 1-reducible as a colimit of the peoples of the various member states, while an "Europe of nations" is a non-reducible object of order 3, because institutionalized links must be mediated by the governments of the states.

A physical example is given by the comparison between the formation of a crystal and of a quasi-crystal. Here the substrate is the HS of chemical products, with atoms and their chemcial links at the level 1. Following Penrose [18], we are going to show that a crystal and a quasi-crystal are level 3 objects, but the first one is 1-reducible, while the second one is not. Indeed, both are constructed in the same two stages: a) Formation of aggregates of atoms according to various motives, represented by level 2 objects. To such an object we associate (as indicated by Penrose) a quantic linear superposition of different arrangements of atoms, which represent its several decompositions. b) In a second step some of these aggregates bind together to form larger conglomerates with a specific topology, and the optimal configuration (the one with the lowest energy) generates the (quasi-) crystal which so emerges at the level 3 (Penrose calls this process the quantic process R). For a quasi-crystal, this second operation cannct be by-passed, for the conglomerate must unite aggregates of several types; hence a quasi-crystal is of 1-order 3. But, a crystal is 1-reducible, because all the aggregates in the conglomerate are of the same nature, without adding other 'horizontal' information, so that the crystal can also be directly described as the colimit of a 'large' pattern of atoms.

6. EMERGENCE OF OBJECTS BY COMPLEXIFICATION

6.1. The Complexification Process [1]

In an ES, a pattern without a colimit may acquire a colimit in time. The emergence of such a colimit actualizes the potentiality of the objects of the pattern to act collectively through their distinguished links. It relies on a 2-fold process: 1. local strengthening of the distinguished links of the pattern to impose stronger constraints and increase cooperation; 2. global emergence of a new 'complex' object which represents the whole pattern as an integrated unit. For instance, during the embryogenesis, a tissue is formed by strengthening a pattern of contiguous cells, via adhesion molecules (CAM, cf. Edelman [14]) which restrain the cells motions; later on it acquires its own functional identity.

This emergence of complex objects will result from successive complexifications. The process of *complexification with respect to a strategy* models the dynamics of natural self-organized systems, which is regulated by the 4 archetypal modifications: birth/death, scission/collision; for instance, in a cell: endocytosis, exocytosis, dissociation and synthesis of macromolecules.

Let **K** be a category representing the state of the system at a instant t. We define a strategy **S** on **K** as follows:

DEFINITION *A strategy* **S** on a category consists in the data of: 1 a set of external objects 'to be absorbed', 2. a set of objects and/or links of the category 'to suppress' (which means they must be mapped on the object 0), 3. a set of patterns without a colimit 'to be binded' so that they acquire a colimit, 4. a set of cones 'to transform into colimit-cones' (their vertex must become a colimit of their base), 5. a set of patterns with a colimit 'to be decomposed' so that the colimit is suppressed.

THEOREM *Given a strategy* **S** *on a category* **K**, *the problem of embedding* **K** *into a category in which the targets of* **S** *are realized has a 'universal' solution, constructed as a functor from* **K** *to a category, called the complexification of* **K** *with respect to the strategy* **S**. (Fig. 8)

In the complexification, the targets of **S** are realized in the most economical way, both on the energetical and algorithmical aspects.

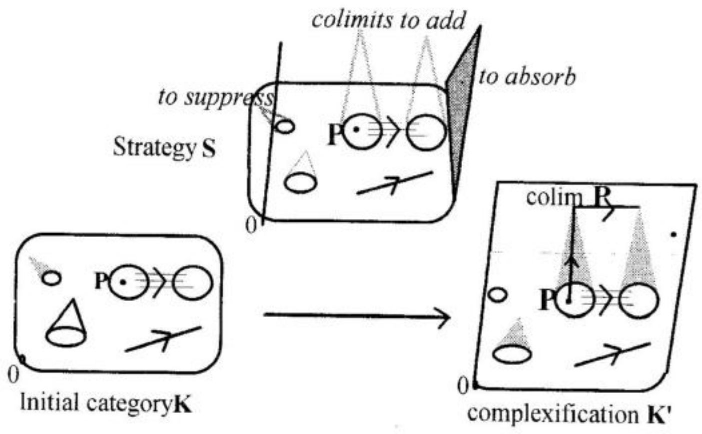

FIGURE 8 Complexification with respect to a strategy.

We now give an explicit construction of this complexification, which is essential to explain the emergence of more and more complex structures.

6.2. Construction of the Complexification [19]

- Its objects are: all the objects of **K** which are not to be suppressed, the objects 'to be absorbed', and, for each pattern **P** 'to be binded', a new object (denoted colim**P**) which will become its colimit and which can be thought of as the pattern itself integrated into a higher level entity with its own identity.
- The links are the links of **K** which are not suppressed, and emerging new links of two types: the simple links obtained by binding clusters of links of **K** between patterns which are required by **S** to have a colimit; and the complex links which are composites of simple links binding non-adjacent clusters through two non-equivalent decompositions of intermediate multifold objects. An object will have non-equivalent decompositions in particular if the strategy requires that it becomes a colimit in the complexification of a certain pattern **Q**′, while it is already the colimit in **K** of a non-equivalent pattern **P**′ to be preserved.

It follows from this construction that, for each pattern **P** to bind, the emerging object called colim**P** becomes really the colimit of the pattern in the complexification. Indeed, each object $\mathbf{P}_i$ of the pattern (considered as the colimit of a pattern with only this object) becomes connected to colim**P** by a 'canonical' simple link which emerges to bind the cluster from $\mathbf{P}_i$ to **P** generated by the identity of $\mathbf{P}_i$. These links form the canonical collective link from **P** to colim**P**. And each collective link from **P** to an object A (new or already in **K**) of the complexification determines a cluster from **P** to the pattern reduced to A, and so it must bind into a simple link from colim**P** to A.

6.3. Iteration of the Complexification

In a natural system, the transition between two successive states is modeled by the process of complexification with respect to an adequate strategy (how the strategy is chosen will be studied later on). Will the iteration of this process during the system evolution lead to the emergence of a hierarchy of more and more complex objects? Or, in other terms, is there only a quantitative difference between the number of emerging objects during a short or a long period, not an increase in their complexity order?

To answer this question, we study the situation after an iteration of the complexification of the category. Let **K**′ be the complexification of the category **K** with respect to the strategy **S** constructed above. This complexification will be considered as a hierarchical system with 2 levels: the level 1 represents the objects coming from the initial category, while the objects of the level 2 are the new colimits of the patterns which the strategy required to bind.

Now we suppose that there is given a strategy **S**′ on the complexification **K**′ and we construct the complexification **K**″ of **K**′ with respect to **S**′. It is a HS with 3 levels, the two first ones coming from the first complexification **K**′ and the objects of level 3 being those which emerge in the second complexification. The problem is to know if an object of **K**″ of level 3 is 1-reducible, i.e., (Section 4) if it can be represented as the colimit of a 'large' pattern of the initial category **K**, or if it 'essentially' necessitates a 2-stages construction.

Let R be a pattern in the first complexification **K**′ which the strategy **S**′ requires to bind; we suppose that each of its objects is of order 2, so that it has emerged in **K**′ to bind a pattern without a colimit in the first category **K**. There must emerge an object A which becomes the colimit of **R** in the second complexification **K**″. We have proved (Section 3) that, if all the links of **R** are simple, this A is 1-reducible, in the sense that it is also the colimit of a 'large" pattern **D** in **K**. But this result may not be true if some of the links of **R** are complex with respect to the level 1, and in this later case the object A colimit of **R** is 'essentially' of order 3. From this we deduce:

THEOREM *If all the links are simple, the objects of the second complexification* **K**″ *are 1-reducible, and* **K**″ *can also be constructed as the (first) complexification of* **K** *with respect to a strategy* **S**″ *on* **K** *uniting all the commands contained in* **S** *and* **S**′. *But if there exist multifold objects, there is no strategy on* **K** *such that* **K**″ *could be obtained after only one complexification.*

Indeed, if all links are simple, each pattern which **S**′ required to bind can be replaced by a large pattern of **K** with the same colimit.

By iteration, we get

COROLLARY *If there are no multifold objects, a sequence of complexifications can be replaced by a unique complexification with respect to an adequate strategy. But this is not possible if there are multifold objects, in which case successive complexifications may lead to the emergence of a hierarchy of objects of strictly increasing orders.*

For instance, in the HS modeling a cell, the synthesis of RNA is obtained by 3 successive complexifications: the first one leads to the primary structure, the second forms the different domains (folding of the loops and helices), then the last complexification gives the complete folding of the tertiary structure. An iteration of complexifications allows the formation of higher order cognitive processes in neural systems (cf. Section 8), or the development of biological systems and of societies during the evolution of the universe.

6.4. Stability Span of an Object

In a natural system, we implicity assume that a component maintains its identity in spite of its internal modifications from its emergence (birth) up to its disparition (death). For instance, a cell perdures though its constituents are progressively renewed; the members of an association change, possibly also its statutes, and yet if the changes are gradual enough, the association will keep its legal identity. Since the objects of an ES are associated to a fixed time, how can we define an 'identity' ('Genidentite' for Helmholtz) between objects in successive state-categories of a HS, specially if their internal organisation varies?

Let B be an object of level $n+1$ in the state-category $\mathbf{K}_t$ at the instant t. The successive states of B are modeled by the images Bt' of B by the transition functors from t to $t'>t$; and the (evolution of the) object is modeled by the *trajectory* ***B*** formed by all these images, up to the time where it becomes 0 (which means that the object is destructed).

At the initial time t, the object B is (by definition of the levels) the colimit in $\mathbf{K}_t$ of at least one pattern **P** of level n. In some cases (e.g., in very stable structures or for short delays), the state Bt' of B at a later date t' will still be the colimit of the new state **P**t' of **P** in the state-category at t'. But from t to t' a number of objects of **P** may be destructed or disconnected from ***B***, while new objects are added; these changes must be gradual enough to maintain some stability.

DEFINITION *The stability span* of ***B*** (or of B) at t is the largest real dt such that there exists a decomposition **Q** of level n of B which remains a decomposition of ***B*** for each s between t and $t+dt$ (in the sense that its image **Q**s is still a decomposition of Bs).

For instance, the stability span of a population of proteins is correlated to its half-life. We say that the change in the internal organisation **P** of B from t to t' is *smooth* if there exists a sequence of decompositions $\mathbf{P}_i$ of states Bt_i, for $t=t_1<\cdots<t_i<t_n=t'$, with $\mathbf{P}=\mathbf{P}_1$ and satisfying the condition: each $\mathbf{P}_i$ has a representative sub-pattern which remains a decomposition of ***B*** up to t_{i+1} and whose image at this date is also a representative sub-pattern which remains a decomposition of B up to t_{i+1} and whose image at this date is also a representative sub-pattern of $\mathbf{P}_{i+1}$ (it entails that the stability span dt_i at t_i is greater than $t_{i+1}-t_i$).

In particular, if B emerges at t to become the colimit of the pattern **P** (e.g., in a complexification with respect to a strategy requiring the binding of **P**), its own evolution and the evolution of **P** remain correlated during the stability span, but after they may progressively diverge; we say that B *takes its own identity*, independent from **P**.

The variation of the stability span gives informations on the rate of change. The span is long during stability periods, while it shortens during periods of development or aging. This decrease is taken as one of the characteristics of aging in the *Theory of aging* through a 'cascade of de-resynchronisations' proposed in EV [7] (where we also introduce the renewal span and the persistance span).

7. RÔLE OF OBSERVERS/ ACTORS IN A MES

The complexification process describes the emergence of objects through a three phases process: a pattern strengthens its distinguished links and so acts as a coherent assembly; this assembly emerges as a higher order object, which in time takes its own identity independent from the initial pattern. But we have not studied how the strategies are chosen. In a natural complex system, the dynamcis is partially controled by internal observers/actors.

7.1. Memory Evolutive Systems (MES)

They have been introduced to model self-regulated natural systems, such as biological or social systems. A MES is an ES whose dynamics is modulated by the interactions between internal control organs, represented by a net of Centers of Regulation (CR) which operate in parallel, each one at its own complexity level and with its own time-scale. Their strategies are cooperative and/or competitive; and the results are memorized to allow for future adaptation. We cannot develop here the study of MES already done elsewere (e.g., in [2]), and we just recall the necessary notions.

DEFINITION A *MES* is a hierarchical ES, with a continuous 'reference' time-scale, in which several evolutive sub-systems are distinguished:

— A hierarchical *Memory* (Mem) which develops in time,
— A net of internal *Centers of Regulation* (CR). Each CR is an evolutive sub-system over its own discrete time-scale formed by a sequence of instants of the reference time-scale; and it operates a stepwise trial-and-error learning process, relying on a differential access to Mem. The objects of a CR, called its *actors*, are objects of a particular level.

The lower level CRs represent specialized modules connected with the environment (such as sensorial organs). In the higher levels, there are associative CRs which supervise several lower CRs. The net of CRs is not purely hierarchical, because there might exist several CRs at the same level, possibly with different time-scales. An example is given by the organigram indicating the various services of an administration.

7.2. Description of One Step of a CR

Each CR operates stepwise, a step extending between two successive instants of its own time-scale. One step, say from t to $t + d$, is divided into several more or less intertwined phases: formation of the landscape of the CR, selection on it of a strategy compatible with the goals, command of this strategy to the effectors, evaluation of the result and its memorization.

1. During the first phase (*actual present* of the CR) the CR, as an observer, collects the informations on the system state which can be available to its actors during the step. The actors have no direct access to the system: an object B is only perceived through its links (or *aspects*) b to the CR, and two aspects b and b' give the same information to the CR if they are correlated by a zig-zag of links between actors (we say that they are in the same *perspective*).

DEFINITION A *perspective* of an object B of the system for the CR is a cluster of the pattern reduced to B to the pattern formed by the CR. It is completely determined by one of its links b. The *actual landscape* [2] of the CR at t is a category whose objects are the perspectives pb of links b which have a propagation delay less than the length d of the step, and which come from objects B of a level near the level of the CR and with a stability span more than d. (Fig. 9)

There exists a functor *distortion* from the landscape to the system, which maps the perspective pb on the object B. The landscape acts as a filter, though it represents the system as it is perceived from the CR. The distortion it introduces with respect to the system cannot be apprehended from inside the landscape. An object or a link *emerges for the CR at t* if it has for the first time a perspective in the landscape at t.

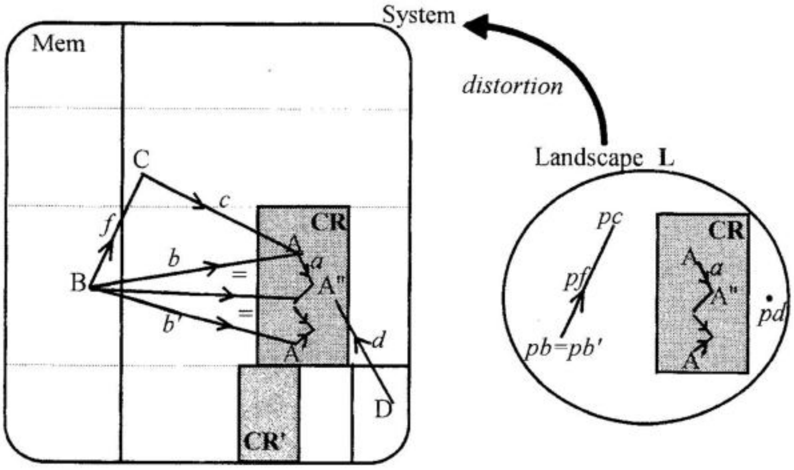

FIGURE 9 The MES and the landscape of a CR.

2. As a regulation organ, the CR evaluates the situation on the landscape and selects an available strategy **S** on it, taking into account the results of the preceding step, the new informations given by the actual landscape, the different constraints. The choice of the strategy may be helped by a recourse to Mem if a similar situation has been memorized, or it can be externally imposed to the actors (for instance by a higher CR). One of the goal of the strategy will be the memorization of the strategy of the preceding step and of its result. (Fig. 10)
3. The CR sends commands to effectors, always through the landscape, to realize the strategy **S**. The anticipated landscape **AL** at the end of the step should be (modeled by) the complexification of the actual landscape with respect to **S**. But the landscape does not give an exact image of the system on which the strategy **S** must be repercuted, and the other CRs also competitively interact on the system. It follows that the goal of the CR are not always realized, and even in some cases the step will be interrupted by a fracture.
4. At the following step, the CR can evaluate the result of the strategy **S** in its new landscape.

THEOREM *There exists a functor comparison between the anticipated landscape* **AL** *and the new landscape formed at* $t + d$. *If this*

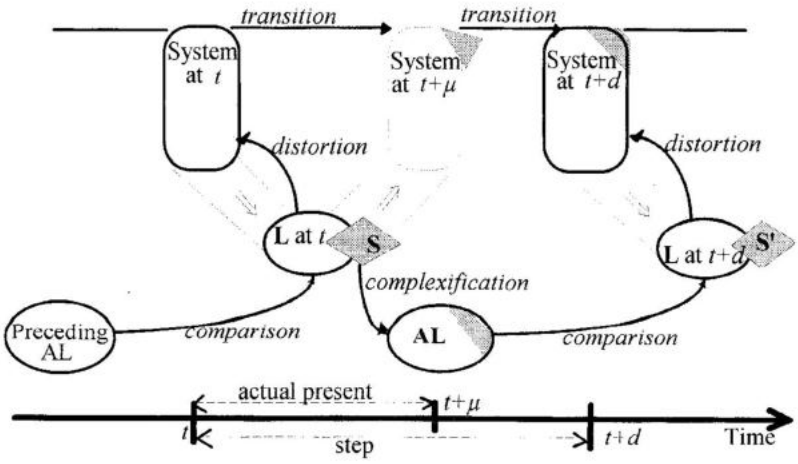

FIGURE 10 One step of the CR.

functor is an isomorphism, the strategy has been realized. Otherwise, it reveals a dysfunction.

7.3. Structural and Temporal Constraints

The dysfunction can be revealed by the emergence in the new landscape of non-anticipated objects, or the absence of some object or link which should figure in the landscape (for instance if the construction of the colimit of a pattern to be binded has not been achieved in time). In simple cases, it can be repaired by an appropriate choice of the next strategy. But in more complicated cases, it may last for several steps, or cause a *fracture* for the CR if there is no available strategy on its landscape, or if the selected strategy cannot be realized. In particular, the following structural temporal constraints (more precisely introduced in EV [7]) must be respected to ensure that the rate of transmission of informations and commands is short enough, and that the internal organisation of the objects considered in the landscape and the strategy remain stable during the step.

THEOREM *(Structural temporal constraints for a CR). The mean length D of the preceding steps of the CR (called its period) must be* *longer than the mean propagation delay p of the links used in the landscape and in the strategy, and less than the mean stability spans s of the objects which are used:*

$$p << D << s$$

These constraints give leave for some plasticity, since they are expressed by large inequalities, not by equalities. If these constraints cannot be respected for several steps, we say that there is a *dyschrony* for the CR.

8. DIALECTICS BETWEEN CRs

In the preceding section, we have considered the operation of a particular CR during one of its steps. Here we examine the global situation on the state-category $\mathbf{K}_T$ of the MES at a time T, resulting from the interactions between the different CRs and from external (e.g., energetical) constraints.

8.1. Repercussion of the CR Strategies to the System

Let CR_i be a particular CR. Since each CR has its own time-scale, T may occur during any of the phases of the step of CR_i; let us consider the case where it is during the command phase (the other phases have less external implications). So CR_i has already formed its actual landscape say $\mathbf{L}_i$, chosen a strategy $\mathbf{S}_i$ on it, and transmitted the corresponding commands to effectors through the landscape. Now the effective realization of the strategy does not concern the landscape (which is only an internal model), but the system itself. This is a first cause of errors, since the commands of $\mathbf{S}_i$ will be transmitted to $\mathbf{K}_T$ with some deformation (via the functor distortion dis_i from $\mathbf{L}_i$ to $\mathbf{K}_T$); for instance if $\mathbf{S}_i$ requires the binding of a pattern $\mathbf{Q}_i$ of the landscape, it translates into the binding of the pattern image of $\mathbf{Q}_i$ by dis_i in $\mathbf{K}_T$. The strategy once repercuted to $\mathbf{K}_T$ will be denoted $\mathbf{S}'_i$.

8.2. Competition Between the Repercuted Strategies

The strategies $\mathbf{S}'_i$ repercuted by the different CRs should all be realized on $\mathbf{K}_T$ for the goals of the CRs to be fulfilled. In this case, the new state of the system should be represented by the complexification of $\mathbf{K}_T$ with respect to a strategy $\mathbf{S}'$ including the various $\mathbf{S}'_i$ and possibly some global commands not transmitted through the CR_i but imposed by external constraints.

Such a strategy $\mathbf{S}'$ would be well determined if the commands of the various strategies $\mathbf{S}'_i$ were compatible, as in the case the different CR_i are strictly parallel and use their own independent resources. But it is not the case in a MES in which the CRs are competing for common resources (be they informations or energy), so that conflicts may arise between two CRs, say between CR_i and CR_j. For instance, in an enterprise several departments depending on a unique repair man may need his help at the same time.

Moreover (and this is essential), the periods of the CRs are different, so that the realization of a strategy may require variable delays. In particular the period of a higher CR_i is much longer than the period of a lower CR_j. Hence during a unique step of CR_i there will be a sequence of successive steps of CR_j each one causing modifications of which CR_i will not be informed in real time because of the propagation delays. These changes may later affect the realization of $\mathbf{S}'_i$, for instance, if this strategy requires to bind together objects some of which have been suppressed because of the intermediate strategies of CR_j.

The results is that there might exist no strategy $\mathbf{S}'$ integrating all the commands repercuted by the different CRs, and in this case the strategy $\mathbf{S}''$ which will be effectively realized on $\mathbf{K}_T$ will not only be a compromise. The formation of $\mathbf{S}''$ results of an equilibration process, called *the interplay among strategies*, through coordination, competition, interferences and compensations between the different strategies of the CRs. This interplay takes into account all the constraints both external (preservation of physical laws) and internal such as the structural temporal constraints of the CRs (cf. Section 7). It also depends on the ponderations of the different strategies of the CRs to minimize the global cost for the system. Analytically, it leads to the formation of attractors of the dynamics.

8.3. Evaluation and Memorization of the Result by the CR

The realization of the strategy $\mathbf{S}''$ may have other consequences for CR_i than those required by the chosen strategy, up to the formation of a fracture in its landscape, $\mathbf{L}_i$. Indeed, $\mathbf{S}''$ may forget some commands of $\mathbf{S}_i$, and add other external commands, thus modifying the landscape of CR_i. The result will be evaluated by CR_i at its next step, by comparison between the anticipated landscape (namely, the complexification $\mathbf{AL}_i$ of $\mathbf{L}_i$ by $\mathbf{S}_i$) and the landscape $\mathbf{L}'_i$ really obtained. The passage from $\mathbf{L}_i$ to $\mathbf{L}'_i$ can be interpreted as the realization of a strategy $\mathbf{S}''_i$ instead of the chosen strategy $\mathbf{S}_i$. If we compare with quantum mechanics, the situation of the system before the interplay among strategies is completed corresponds to a superposition of states, and the result of the interplay (evaluated in a CR) to the statevector reduction after a measurement.

One of the goals of the strategy $\mathbf{S}_i$ of CR_i is to memorize the preceding strategy and its result, so that it might be re-used if a similar situation recurs. The strategy should be stored as a higher level object m_i binding its several commands of effectors. But after passage to $\mathbf{S}''$, it will be represented by an object M (in the central memory Mem) corresponding to the global memorization of all the objects repercuting the various (perspectives) m_i to the system through dis_i. And the later recall of m_i will consist in the activation of a perspective m'_i of M in the landscape of CR_i. Whence a double cause of errors, in the formation of M and in its recall 'as' m'_i. But also some plasticity in the choice of the strategies since the recall of the higher level object M may be realized through anyone of its ramifications, depending on the constraints coming from the interactions between the CRs. For instance, a general command such as 'take an object on a table' will activate different muscles depending on the size and conformation of the object.

Finally, we have

THEOREM *The dynamcis of the MES is the consequence of a double process: 1. the local choices of a strategy* $\mathbf{S}_i$ *by each* CR_i*; 2. the ensuing interplay among (repercuted) strategies which generates the global strategy* $\mathbf{S}''$*, later evaluated as* $\mathbf{S}''_i$ *in the landscape (via the comparison functor) and memorized with its result. Each phase of the passage from* $\mathbf{S}_i$ *to* $\mathbf{S}''_i$ *depends on internal and external constraints and may introduce some deformation with a risk of fracture.*

8.4. Classification Process

The memorization of more and more complex strategies and of their results leads to the development of a *procedural Memory*, differentially used by the various CRs to select their strategies. In complex enough MES, a classification process will develop to memorize classes of items (e.g. of strategies) having some common features, thus confering still more plasticity to the recall operation. This classification relies on the formation of (projective) limits [9] instead of colimits (= inductive limits).

Indeed, the colimit of a pattern internalizes its capacity for collective actions on other objects. But the objects of a pattern can also cooperate to collectively decode messages of which they individually receive only a part. This potentiality will be actualized by the formation of a *limit* of the pattern.

DEFINITION The *limit* L of a pattern is formally defined as the colimit, except that all the links are inverted (Fig. 11). The set of objects C with a link toward L is called the *invariance class classified* by L.

The invariance class is formed by the objects which send a *common message* to the pattern, and this common message is decoded through the limit. So the limit 'classifies' those objects which possess the characteristic features decoded by the pattern.

All we have said about colimits transposes to limits. The construction of the complexification described in Section 6, with simple and

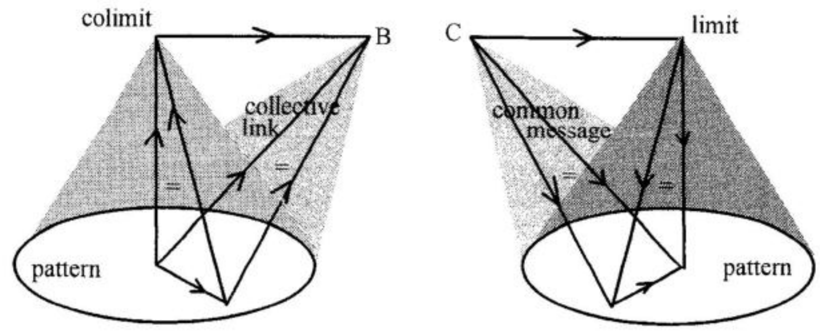

FIGURE 11 Limit and colimit of a pattern.

complex links, extends to the case in which the strategy requires also that some patterns 'be classified' (so that a limit be added to them). And by iteration we also construct iterated limits, with several downward ramifications. In particular, the formation of limits classifying items in the memory may lead to the development of a *semantical memory*.

Such a memory adds flexibility to the interplay among strategies. Indeed, in a higher CR, say CR_i, the choice of a strategy S_i will be done through the semantical memory, under the form of a limit classifying an invariance class of strategies, instead of selecting a particular strategy of the class. Thus we have a new liberty degree concerning the formation of the 'effective' strategy S'', to activate the best adapted instance of the invariance class depending on the strategies repercuted by the other CRs (cf. EV [21]). It gives a double freedom to fulfill the slots of the 'frame' M which memorizes the strategy, first thanks to the choice of a particular ramification of M, and secondly through the choice of specific instances of the invariant classes classified by its different components.

8.5. Dialectics Between Heterogeneous CRs

The question which remains is: which commands of S' will be performed? Or still in the competition between the CRs, which are the winners? There is no general answer, for it depends on the structure of the system and of its net of CRs. For instance, if the CRs form a command hierarchy (e.g., in an army), the higher levels will impose long term strategies on the lower levels, while there is much more flexibility in parallel distributed systems, except if there exists a higher central 'executive' CR which imposes a priority order on the commands of the other parallel CRs.

Generally in self-organized natural systems, the strategies of the lower CRs are pre-eminent on the short term, but these CRs are controled by higher CRs with longer periods which may impose their strategies on the long term, either to disentangle on obstruction, or to avoid or repair a fracture at their own level. For example, when the DNA of bacteria has been much damaged, the replication process is halted because usual simple macromolecular repair mechanisms are outflowed; then the cell level may activate the SOS system (Radman

[20]) so that the SOS products emerge in the macromolecular landscape and impose a strategy to carry on the replication even if without a strict pairing of the bases (possibly leading to a mutation).

Anyway, there remains some plasticity, hence some unpredictibility, because of the multiple ramifications of a higher level object. We have shown [2] how this process leads to a *dialectics between heterogeneous* CRs with differing levels and time-scales. It characterizes the behavior of a complex system and shows that any approximation (or simulation) by a 'simple' physical system is only valid "locally and temporarily" (as suggested by Rosen [4]).

Indeed, usual models (e.g., dynamical systems) could describe one particular complexification process because it is entirely determined by the initial state and the strategy (which gives the parameters). But they could not describe a non-reducible sequence of complexifications, in which new objects emerge at each step (cf. Section 6). In terms of the Aristoteles causes, we can say that usual models can be applied when the material, formal and efficient causes are well defined (here, by the initial state and the strategy), but in a sequence of complexifications these causes are entangled, with a gradual unfolding of the material cause on which depends the unfolding of successive formal and efficient causes (to be compared with the unfolding of Bohm's implicit order [22]). Moreover even if we knew the strategies of the CRs, we could not deduce the result of the interplay among strategies, which depends on an adjustment between the different internal and external constraints and the structural temporal constraints of the CRs.

9. APPLICATION TO NEURAL SYSTEMS

The MES modeling a neural system is formed by successive complexifications from the category of neurons (defined in Section 1), which lead to the formation of higher order objects which we call *category-neurons*. This MES satisfies the Multiplicity Principle.

9.1. Category-Neurons

In the category of neurons, a pattern of neurons linked by synaptic paths has a colimit only if there exists a specific neuron N which 'personifies' the assembly, in the sense that the activation of N has the same effects as that of the whole pattern; in this case, N is called its *pilot-neuron*. For instance, in the visual areas, there exist such a pilot-neuron representing (the group of neurons excited by) edges (Hubel & Wiesel [23]), or, a monkey has a pilot-neuron representing a hand holding a banana [24].

But generally a pattern has no pilot-neuron (no 'grand-mother neuron'). However some patterns without a pilot-neuron may, in time, reinforce their cohesion to form a synchronous assembly in Hebb's sense [25], or a neural group in the terminology of Edelman [14]. In our MES, these are represented by 'abstract' higher order objects, called *category-neurons*, introduced through successive complexifications.

A category-neuron of order 2 emerges as the colimit of a pattern of neurons which has no pilot-neuron, but acts as a synchronous coherent assembly of neurons in the sense of Hebb. A category-neuron of order 3 corresponds to a super-assembly (or 'assembly of assemblies') of neurons, and it cannot be reduced to a (large) synchronous assembly of simple neurons if some of the distinguished links of the pattern of category-neurons of level 2 which it binds are complex. Higher category-neurons in successive complexifications correspond to super-super-assemblies, and so on. Moreover the possible interactions between the category-neurons are well described, since they are the simple and complex links of successive complexifications. So it becomes possible to 'compute' with these category-neurons as if they were simple neurons, thus developing a real "algebra of mental objects" (following the proposition of Changeux [26]).

Let us remark that this model is very different from neo-connexionnist models of neural systems [27] which give only a description at the sub-symbolic level, and for a limited period, without taking into account the interactions between the different levels. In particular, these models can only describe the formation of category-neurons of order 2 (represented by attractors of the dynamics) but not of higher order neurons.

9.2. Development of a Semantical Memory

The successive complexifications can also lead to the development of a classification process, with formation of limits classifying invariance classes of items (e.g., of strategies). These emergent limits, which we call *concepts*, become objects of a semantical memory (cf. Section 8).

A concept is formed in two phases (cf. EV [21]): first 'practical' differentiation of a pattern of lower actors activated by a specific type of messages (for instance, in a colour-CR, the pattern of receptors activated by blue objects), then its classification as a 'CR-concept' (the concept 'blue') memorized (by a higher level CR) as the limit of this pattern. This limit classifies the invariance class of the concept (all the images of blue objects). The CR-concepts form a Semantical Memory, which is extended by the formation of more abstract concepts as colimits of patterns of such 'concrete' concepts linked by complex links. As we have said before (Section 8), the development of such a semantics adds flexibiltiy to the choice of strategies and their interplay, and it will be used by higher CRs with particular properties (possibly emerging in successive complexifications).

9.3. Intentional and Conscious CRs

A CR will be called a *D-intentional* CR, or intentional system in the sense of Dennett [28], if it acts 'as if' it was able to optimize its choice of strategy in its landscape. This is possible if some of its actors (at least two, say + and −) are *evaluators* which classify invariance classes of already memorized strategies depending on their result; for instance, + classifies successful strategies. A comparison between strategies classified by the same evaluator can be done by comparing the weights of the links from the strategies to the evaluator.

If the strategies are internalized in the landscape by the limit of the pattern of effectors which they activate, the comparison may also be internalized in the landscape; and we say that the CR is *intentional*. This property requires the formation of loops between the CR and the procedural and semantical memories.

The *conscious* CRs are particular intentional CRs, able to internalize the notion of time. Let us recall [21] that they are characterized by the capacity, after a fracture, to extend their actual landscape by retrospection to research the causes of the fracture, and by projection in the future to chose strategies for several steps ahead. Their existence relies on the existence of functional loops between various areas of the cortex, which form what Edelman [14] calls a 'loop of consciousness'.

9.4 Brain/Mind Problem

The representation of a mental state, such as a complex cognitive process, by a category-neuron of a higher order leads to a new approach of the philosophical problem of the *identity between mental states and physical states of the brain.* Indeed, a physical state, as it is seen through brain imagery, corresponds only to the activation of a simple assembly of neurons (i.e., a category-neuron of level 2). But a higher order category is non-reducible to a simple assembly to neurons, though it is constructed by successive complexifications from the neuron level, and has ramifications down to this level. So its activation requires a several steps unfolding through the various intermediate levels of a ramification, down to the level 2 of physical states; and at each step, it can proceed along one or another non-equivalent decomposition of multifold objects, with possibly a switch between them, whether of a random origin (neural 'noise') or controlled. Though such a process represents a well described physical 'event', we cannot identify it with a physical 'state': *mental states emerge in a dynamical way (through the gradual unfolding of a ramification) from physical states but are not identical to them.* This could define an *emergentist monism* in the sense of Bunge [13]. Could we follow Eccles [29] and consider this switch as resulting from a quantic mechanism by which the mental could act on the physical states? We think that this dualist interpretation is contrary to the constructibility of mental states described above.

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