Des espèces de structures locales aux distructures et systèmes guidables

by

Andrée C. Ehresmann
Université de Picardie Jules Verne
http://pagesperso-orange.fr/ehres
http://pagesperso-orange.fr/vbm-ehr

ENS, Juin 2011

FIRST PUBLICATION OF JEAN BENABOU

Faculté des Sciences de Paris 2-01

-:-:-:-

20 et 27 novembre 1957
et 11 décembre 1957

Séminaire de TOPOLOGIE
et de GÉOMETRIE DIFFÉRENTIELLE
(C. EHRESMANN)

Année 1957/1958

-:-:-:-

TREILLIS LOCAUX ET PARATOPOLOGIES

par Jean BENABOU

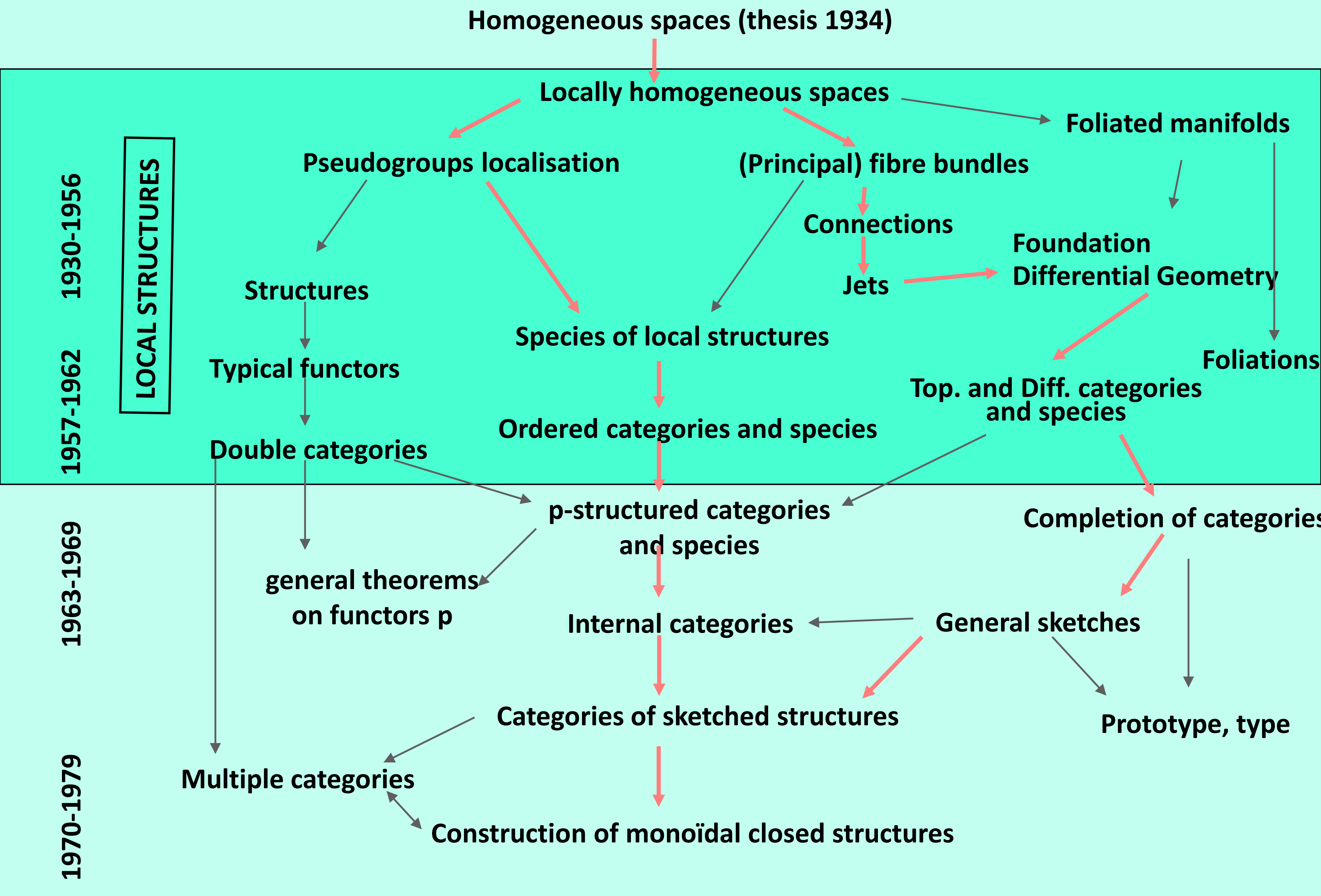
Section I.

Notions élémentaires sur les paratopologies

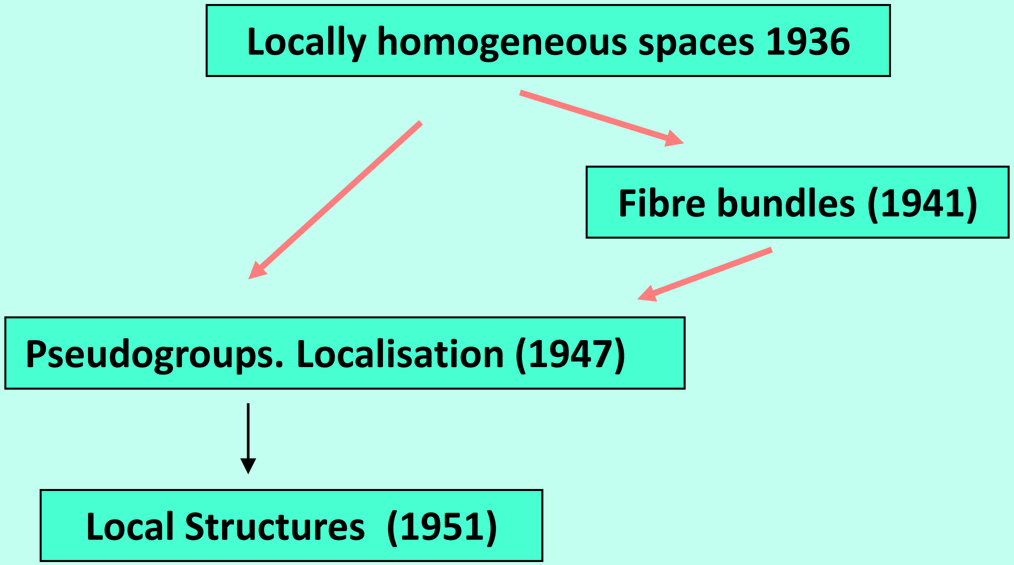
1. Introduction

I. ON SPECIES OF LOCAL STRUCTURES

DIAGRAM OF CHARLES EHRESMANN'S WORKS



INTRODUCTION OF LOCAL STRUCTURES



Charles' work on homogeneous spaces (1934) is extended to *locally homogeneous spaces* (1936), that is spaces in which each point has an open neighbourhood isomorphic to a given homogeneous space.

His work on fibre bundles gives other examples of "local structures", thus raising the general problem of "localisation" for which he introduces in 1947 the notion of a *pseudogroup of transformations* (refining a notion of Veblen-Whitehead) and constructs its associated *local structures*.

PSEUDOGROUP OF TRANSFORMATIONS AND ASSOCIATED LOCAL STRUCTURES

Extract from "Sur la théorie des espaces fibrés"
Colloque Paris, CNRS 1947

n'admet pas de structure presque complexe. Par contre, la sphère S_6, et plus généralement toute variété différentiable orientable à six dimensions dont le groupe d'homologie de dimension 3 est nul, admet une structure presque complexe.

Pour finir je considérerai un problème analogue concernant les variétés analytiques complexes V_{2n}, à $2n$ dimensions complexes, ce qui conduit à une notion de *variété presque quaternionienne*.

Les nombres placés entre crochets renvoient à l'index bibliographique; celui-ci n'est pas une liste complète des publications concernant les questions traitées.

1. Définition d'une structure compatible avec un pseudo-groupe de transformations (¹). — Soient M un espace topologique, et Φ un ensemble d'ensembles ouverts de M tel que toute réunion et toute intersection finie d'ensembles de Φ appartiennent à Φ. Soit Γ un ensemble d'homéomorphismes vérifiant les axiomes suivants :

1° Tout homéomorphisme $f \in \Gamma$ est défini dans un ensemble $U \in \Phi$ et l'on a $f(U) \in \Phi$.

2° Soit U la réunion d'une famille d'ensembles U_i appartenant à Φ. Pour qu'un homéomorphisme f défini dans U appartienne à Γ, il faut et il suffit que sa restriction à U_i appartienne à Γ.

3° Pour tout $U \in \Phi$, l'application identique de U appartient à Γ. Si $f \in \Gamma$, l'application réciproque f^{-1} appartient à Γ. Si f et f' sont deux homéomorphismes appartenant à Γ, tels que le composé ff' soit défini, alors ff' appartient à Γ.

L'ensemble Γ vérifiant ces axiomes sera appelé *pseudo-groupe de transformations*.

Exemple. — Les homéomorphismes différentiables dont chacun transforme un ouvert de R^n en un ouvert de R^n forment un pseudo-groupe de transformations dans R^n.

Soit E un deuxième espace topologique. Nous appellerons *carte*

(¹) Nous reprenons ici, en les précisant, les notions introduites pour la première fois par O. Veblen et J. H. C. Whitehead dans *The foundations of differential geometry*, Cambridge Tracts, 1932.

In this definition of a *pseudogroup of transformations*, Charles does not indicate that it is a groupoid, a notion he will use in his 1950 paper on connections.

He introduces the *associated local structures*, by gluing the charts of an atlas in which the changes of charts belong to the pseudogroup :

The topological space E is *locally isomorphic* to M with respect to the pseudogroup of transformations Γ on M if there is an atlas A = (c_i) such that E is the union of the domains U_i of its charts c_i and all the changes of charts $g = c_j c_i^{-1}$ are in Γ, meaning that

$$c_j(x) = g(c_i(x))$$

for each x in the intersection of U_i and U_j.

SPECIES OF LOCAL STRUCTURES
1952

GÉOMÉTRIE DIFFÉRENTIELLE. — *Structures locales et structures infinitésimales.*

Note de M. Charles Ehresmann, présentée par M. Arnaud Denjoy.

Cette Note fait suite à trois Notes antérieures (1) et résume un exposé fait à la Réunion des Mathématiciens du Rhin Supérieur à Bâle le 15 décembre 1951. Relations entre la notion d'espèce de structures locales et celle de pseudogroupe de transformations. Définition d'une structure infinitésimale. Groupoïde et pseudogroupe associés à une structure infinitésimale. Pseudogroupe de Lie.

1. Une *espèce de structures locales* est une espèce (2) de structures (α) pour laquelle il existe une *loi d'induction*, c'est-à-dire une loi qui associe à toute structure $\mathfrak{S}$ d'espèce (α), donnée sur un ensemble E, un ensemble Φ de parties de E et qui détermine sur tout ensemble $U \in \Phi$ une structure d'espèce (α) appelée *structure induite* par $\mathfrak{S}$ sur U, l'ensemble U muni de cette structure induite étant appelé *sous-espace distingué* de E, de telle façon que les conditions suivantes soient satisfaites :

1° Φ est l'ensemble des ensembles ouverts d'une topologie sur E. On dira que $\mathfrak{S}$ est une structure locale par rapport à cette topologie.

2° *Transitivité des structures induites :* Si U est un sous-espace distingué de E, les sous-espaces distingués de U sont les sous-espaces distingués de E qui sont contenus dans U.

3° Si M est la réunion d'une famille d'ensembles M_i dont chacun est muni d'une structure d'espèce (α) telle que M_i et M_j de la famille admettent $M_i \cap M_j$ comme sous-espace distingué à structure induite bien déterminée, il existe sur M une structure d'espèce (α) bien déterminée telle que chaque M_i soit sous-espace distingué de M.

L'ensemble des *automorphismes locaux* de E, c'est-à-dire des isomorphismes d'un sous-espace distingué sur un sous-espace distingué, est un *pseudogroupe de transformations* Γ. Inversement, étant donné un pseudogroupe de transfor-

(1) *Comptes rendus*, **233**, 1951, p. 598, 777 et 1081.

(2) N. Bourbaki, *Théorie des Ensembles* (fascicule de Résultats), Paris, (Hermann).

This Note to CRAS (February 1952) is the first publication with a formal definition of a *species of local structures*.

It has been developed n the contemporary paper:

"Structures locales" (Conference Rome, 1952).

1957: "GATTUNGEN VON LOKALEN STRUKTUREN"

Ehresmann, Charles
Jahresbericht d. DMV
Bd. 60 (1957) S. 49—77

Gattungen von lokalen Strukturen.

Von CHARLES EHRESMANN in Paris.

In dieser Arbeit[1]) wird der allgemeine Begriff einer Gattung von mathematischen Strukturen entwickelt, ausgehend von den Begriffen „Kategorie“ und „Funktor“, die von Eilenberg-Mac Lane ein- 1
geführt worden sind. Dies führt besonders zu einer Theorie der Gattungen von lokalen Strukturen[2]). Die Mannigfaltigkeiten der Kategorie C^r, die lokalen Produkte, die Blätterungen und die Faserungen bilden wichtige Gattungen dieser Art. Die Beweise sind bloß kurz angedeutet, da sie leicht ergänzt werden können. Die Grundstrukturen der Differentialgeometrie sollen in einer Fortsetzung zu dieser Arbeit be- 2
handelt werden.

Wir unterscheiden zwischen Mengen und Klassen; eine Menge ist auch eine Klasse. Die Klasse aller Mengen ist keine Menge. Wir lassen

---

1) Diese Arbeit ist die Darstellung des ersten Teils meines Vortrags über „Grundbegriffe der Differentialgeometrie“, gehalten bei der Tagung der Deutschen Mathematikervereinigung in Würzburg (September 1956). Der zweite Teil dieses Vortrags brachte eine Definition der Grundstrukturen der Differentialgeometrie. Er wird in einer Fortsetzung zu dieser Arbeit erscheinen.

2) Die Grundideen dieser Theorie habe ich schon in früheren Publikationen[3]) kurz angegeben und seit 1952 in verschiedenen Vorlesungen und Vorträgen ausführlich mit Anwendungen dargestellt (z. B. in Rio de Janeiro, Princeton, Yale, Bombay, Paris). 3

3) a) Structures locales et structures infinitésimales (Comptes Rendus Acad. Sciences, Paris, 234, 1952, p. 587).
b) Structures locales (Annali di Mat., 1954, p. 133).
c) Introduction à la théorie des structures infinitésimales et des pseudogroupes de Lie (Colloque Int. de Géométrie diff. de Strasbourg, C.N.R.S., 1953).

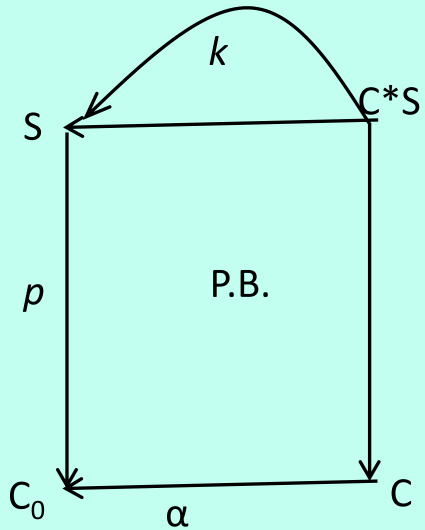
125

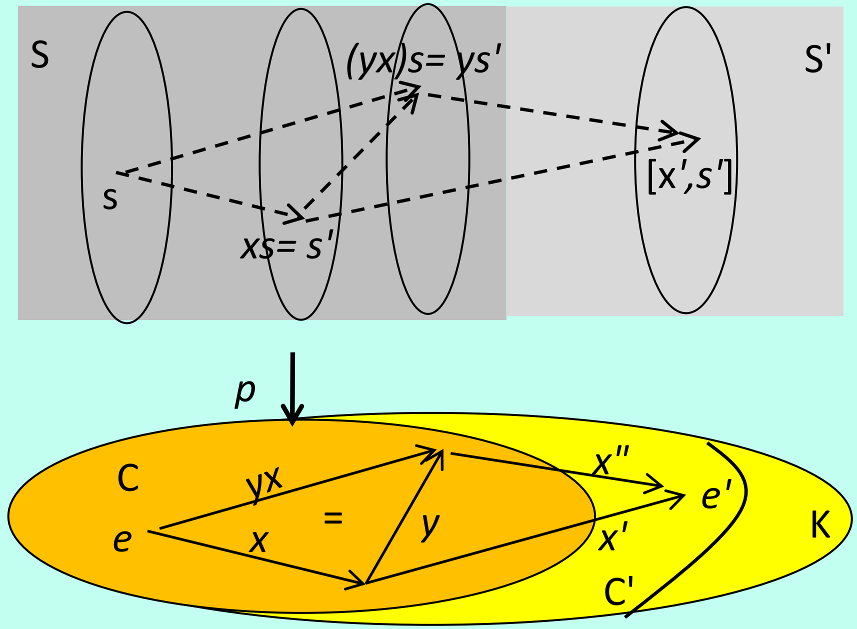
First paper in which Charles uses the term "category" (though he had often used "groupoid" since 1950).
It formalizes the definition of a *local structure*.

The main insight is that in a pseudogroup of transformations on M, the "points" of M are not useful, only the open sets of its topology. Thus the topology can be replaced by a "topology without points" or "*paratopology*", a notion which has become extensively used in the seventies under the name of "locales".

The ideas in this paper have paved the way for a large number of subsequent papers, e.g., on ordered categ-ories, internal categories and completion theorems.

SPECIES OF STRUCTURES





A *species of structures* Σ onto a category C consists of a set S, a map *p* from S onto C_0 and a composition *k*: C*S → S: (*x, s*) |→ *xs,* where C*S = {(x, s)| x є C, *s* є S, α(*x*) = *p(s)*}, satisfying:

y(xs) = (yx)s and α(*x*)*s* = *s*.

Charles also says that *k* is an *action of* C *on* S. And he constructs the associated *discrete fibration*.

Enlargement Theorem. *If C is a subcategory of* K*, the species* Σ *is 'universally' extended into a species onto the full subcategory* C' *of* K *whose objects e' are the target of at least one x' with its source in* C_0.

An element of the fibre over *e'* is an *atlas* consisting in an equivalence class [*x', s'*] of *cards* (*x', s'*), with *s' є S* and x': *p(s')* → *e',* for the equivalence generated by: (x"y, s') ≈ (x", *ys'*).

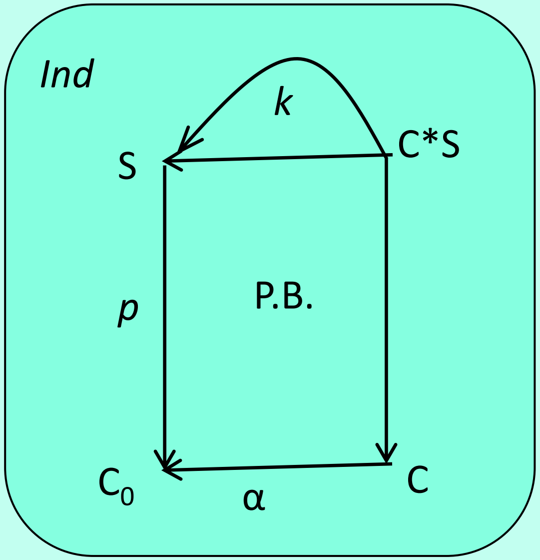
LOCAL SPECIES OF STRUCTURES

A *local class* is a poset in which each bounded part A has an aggregate VA (= lubA) and which satisfies the *distributivity axiom*:

$$(\mathrm{V}A) \wedge b) = \mathrm{V}_{a\epsilon A}(a \wedge b)) \quad \text{for each bounded part A.}$$

An *inductive map* between posets respects all the aggregates and the intersection of a finite bounded part. Inductive maps between local classes form the category *Ind.*

A *local category* is a category internal to the category *Ind* such that the order induced on the Hom is discrete.



A *local species of structures* over a local category C is an Internal to *Ind* species of structures (S, *p*, C), that is S is local and the maps

$$p\colon S \to C_0 \quad \text{and} \quad k\colon C^*S \to S$$

are inductive.

Then the associated fibration is also a local category.

Local Enlargement Theorem. *If Σ is a local species of structures onto a subcategory* C *of a local category* K*, its enlargement is a local species of structures*.

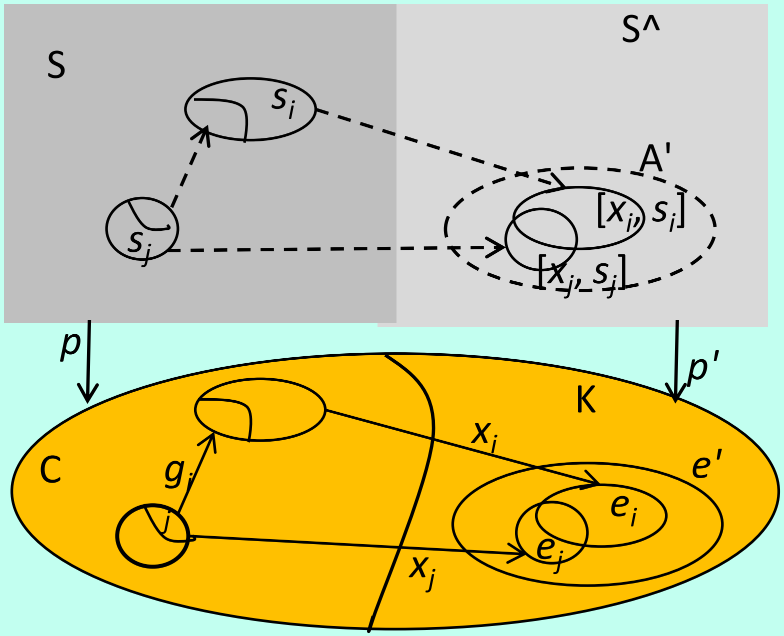
COMPLETE ENLARGEMENT THEOREM

A local species of structures is *complete* if each *p*-compatible part A of S such that *p*(A) is bounded admits an aggregate, where *p*-compatible means that the restriction of *p* to A respects intersections. (Kind of "sheaf condition", but without invoking points.)

Theorem. *Let* Σ *be a local species of structures onto a local subcategory* C *of* K*.*

1. ***Completion*** (= Sheafification). Σ *admits a universal embedding into a complete local species of structures onto* C*.*

2. ***Complete Enlargement.*** Σ *admits the completion of its enlargement as a universal embedding into a largest complete local species of structures*.



An element of the completion is a maximal *p*-compatible class A such that V*p*(A) exists.

An element of the fibre of the complete enlargement over *e'* is an "*atlas*" uniting the atlases $[x_i, s_i]$ of a *p'*-compatible class A' of the enlargement of Σ such that *e'* = V*p'*(A').

Remark. A local category is *completely regular* if for objects *e* < *e'* there exists

e'e: e → e' such that e < *e'e* < *e'*.

Then the *e'e* form a Grothendieck topology J and the complete enlargement compares to the associated sheaf for J.

II. SEMI-SHEAVES
ASSOCIATED PRESHEAF AND SHEAF

DISTRIBUTIONS AS LOCAL STRUCTURES

The *distructures* (introduced in my thesis, 1962 and translated in a modern language in the Calais Conference 2008) give a categorical frame for studying different kinds of "generalized functions", and in particular for extending Schwartz vector distributions to infinite dimensional locally convex vector spaces (lcs).

" Une distribution est, localement, une dérivée d'une fonction continue "
Laurent Schwartz, *Théorie des distributions* (1950), p. 8

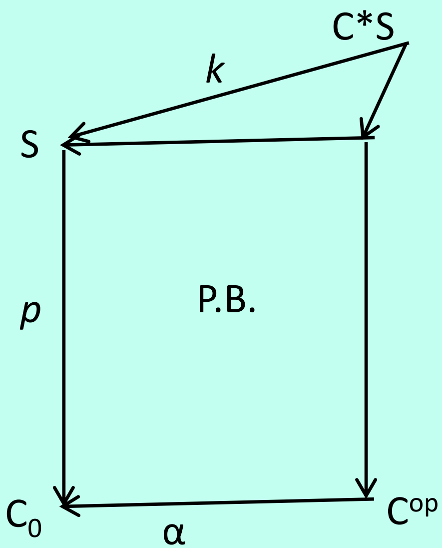
Imitating Charles' Complete Enlargement Theorem, the above citation suggested to construct distributions on a lcs E, with values in an lcs E' through a "di-completion" process:

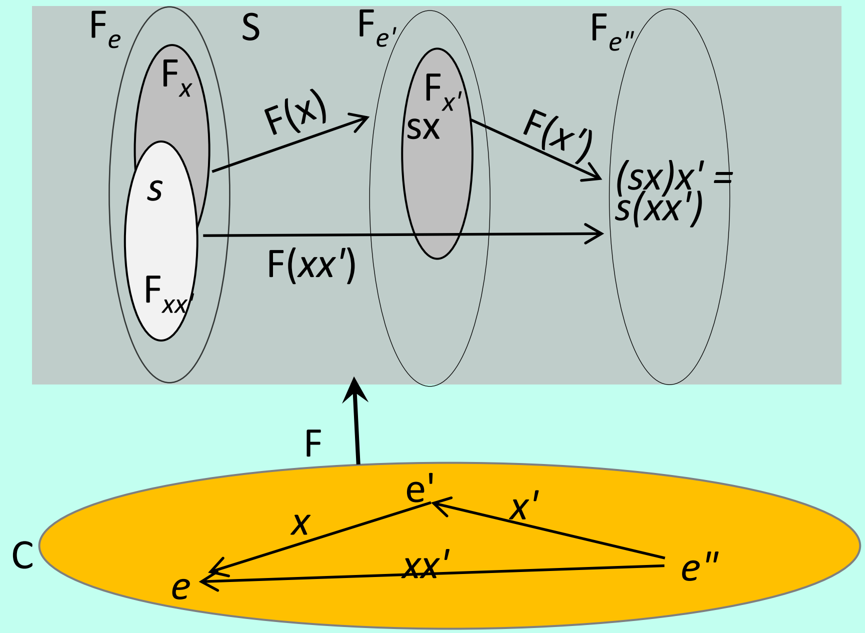
(i) Differential operators *d* define a 'partial action' on the lcs of continuous maps from an open subset U of E to E', the composite *df* being defined only if *f* has a continuous *d*-derivative. This partial action is extended into an action (enriched in Lcs), leading to the presheaf of finite order E'-valued distributions over E.

(ii) A sheafification of this presheaf leads to the sheaf of E'-valued distributions .

The first step leads to define a partial action of a category, or (seen from bottom to top) a *semi-sheaf* on the opposite category.

SEMI-SHEAVES OF SETS



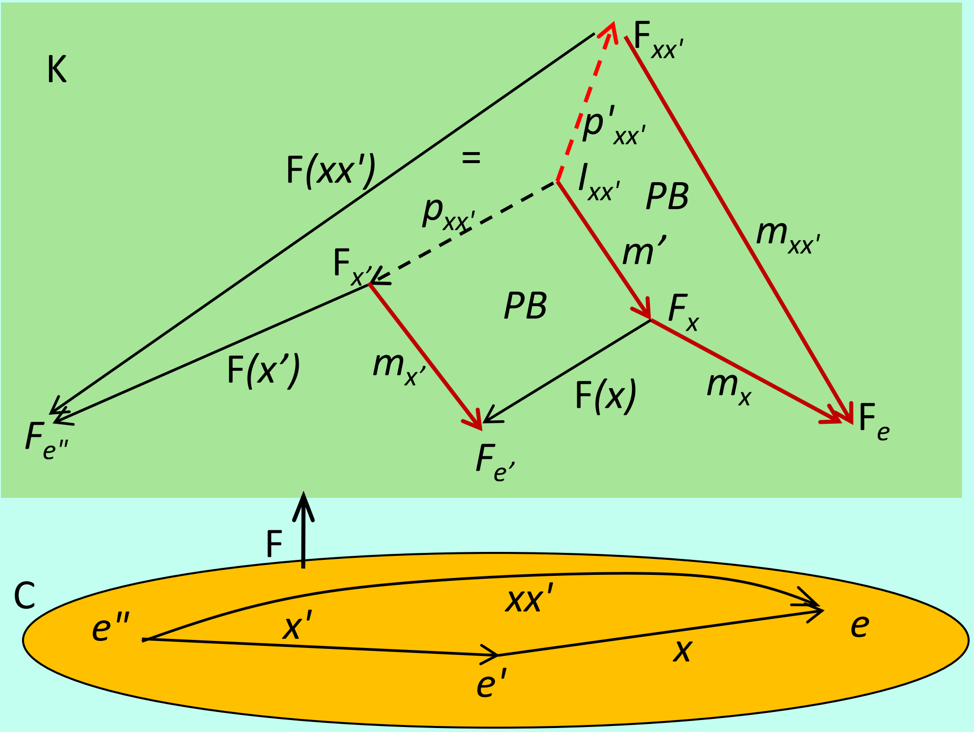


A *semi-sheaf (of sets)* F on C is a map from C to *Set* which associates to an object e of C the *fibre* F_e on e, to an arrow x from e' to e a map $F(x)$ from a sub-set F_x of F_e to the fibre on e', and satisfies the 'transitivity condition':

If s is in F_x then $sx = F(x)(s)$ is in $F_{x'}$ if and only if s is also in $F_{xx'}$, and then $s(xx') = (sx)x'$.

We also say that the map k: $(x, s) \mid\rightarrow sx$ if s is in F_x defines a *semi-action* of C^{op} on the disjoint union S of the fibers, or that S is a *system of structures* onto C^{op}.

(K, M)-**SEMI-SHEAVES**



K is a category and M a sub-category of monomorphisms (in red) of K containing the identities, stable by pullbacks and with at most one *m* between 2 objects.

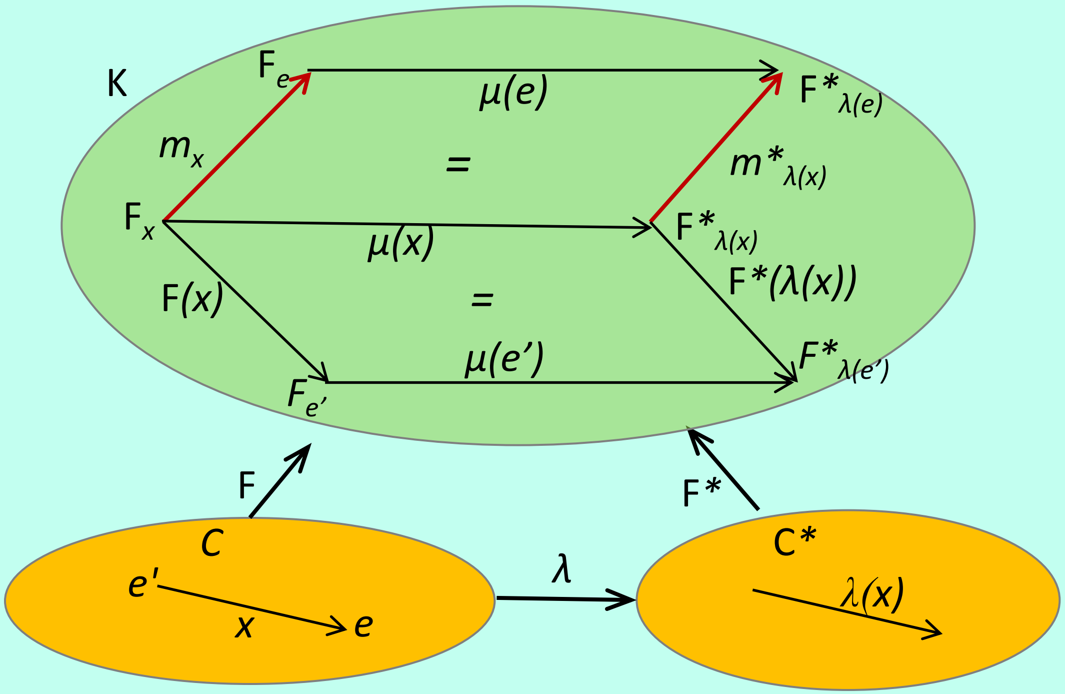
A (K, M)-*semi-sheaf* F on C is a map from C to K such that:

1. F(*e*) is an object F_e for each object e of C.

2. For $x: e' \to e$, F(*x*): $F_x \to F_{e'}$ where F_x is an object such that there exists an m_x: $F_x \to F_e$ in M.

3. '*Transitivity condition*': for $x': e'' \to e'$ in C, the pullback $I_{xx'}$ of $(m_{x'}, F(x))$ is also the pullback of $(m_x, m_{xx'})$ and $F(xx')p_{xx'} = F(x')p'_{xx'}$ where $p_{xx'}$ and $p'_{xx'}$ are projections of the PB's.

THE CATEGORY SS(K, M)

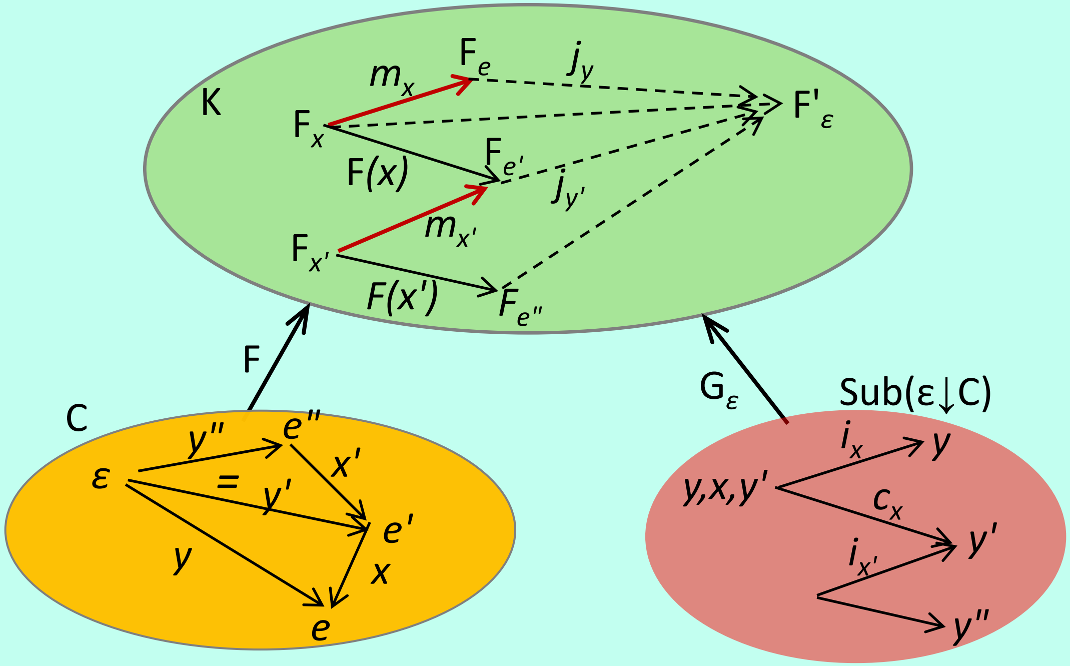


If F is a (K, M)-semi-sheaf on C and F* a (K, M)-semi-sheaf on C*, we define a *morphism* from F to F* as a couple (λ, μ): F → F* of a functor λ from C to C* and a map μ from C to K making commutative the above diagrams.

The category SS(K, M) admits as a full subcategory the category Diag(K) of K-presheaves and has a functor Base toward Cat.

Examples. Particular (Top, open ins)-semi-sheaves, called *nuclei of actions*, are used to study control problems for differential equations. The Evolutive Systems (defined with J.P. Vanbremeersch for modeling living systems) are (Cat, subcat)-semi-sheaves.

K-PRESHEAFIFICATION OF A (K, M)-SEMI-SHEAF

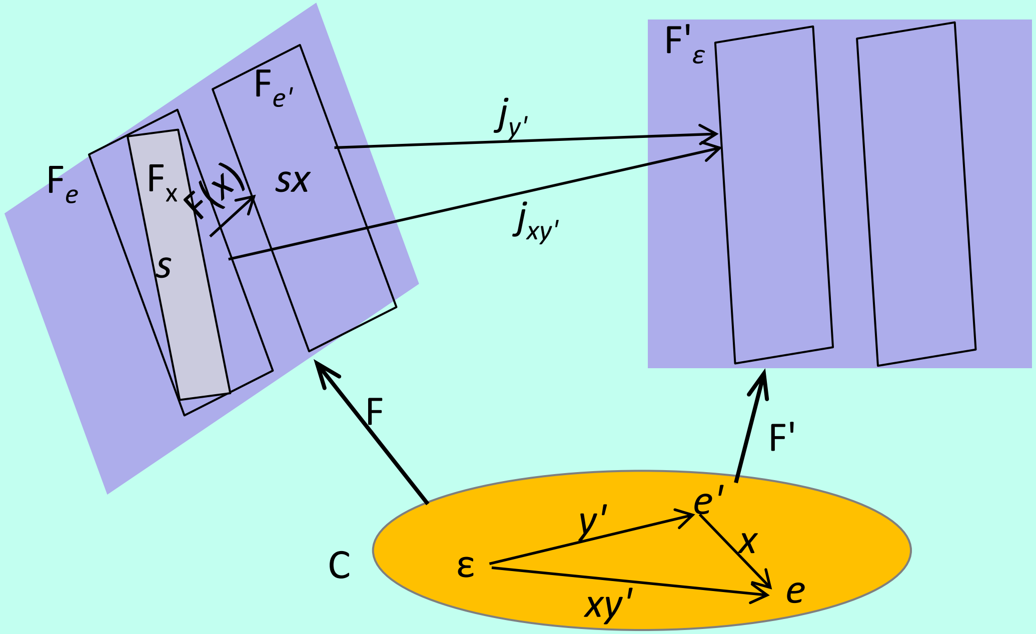


Associated Presheaf Theorem. *If* K *admits pullbacks and enough colimits, the category* Diag(K) *of* K*-presheaves is a reflective subcategory of the category* SS(K, M) *of* (K, M)*-semi-sheaves.*

The K-presheaf F': $C^{op} \to$ K associated to the (K, M)-semi-sheaf F is explicitly constructed: its fiber F'_ε is the colimit of the functor G_ε: Sub(ε↓C) → K where Sub(ε↓C) is the subdivision category of the category ε↓C of objects under ε,

$$G_\varepsilon(i_x) = m_x \quad \text{and} \quad G_\varepsilon(c_x) = F(x).$$

LOCALLY CONVEX SEMI-SHEAVES



If K is the category Lcs of locally convex spaces and M_V the class of insertions of a vector subspace with a finer topology, a (Lcs, M_V)-semi-sheaf F is called a *locally convex semi-sheaf,* and its associated Lcs-presheaf F' a l*ocally convex presheaf*. If the fibers of F are complete, so are those of F'.

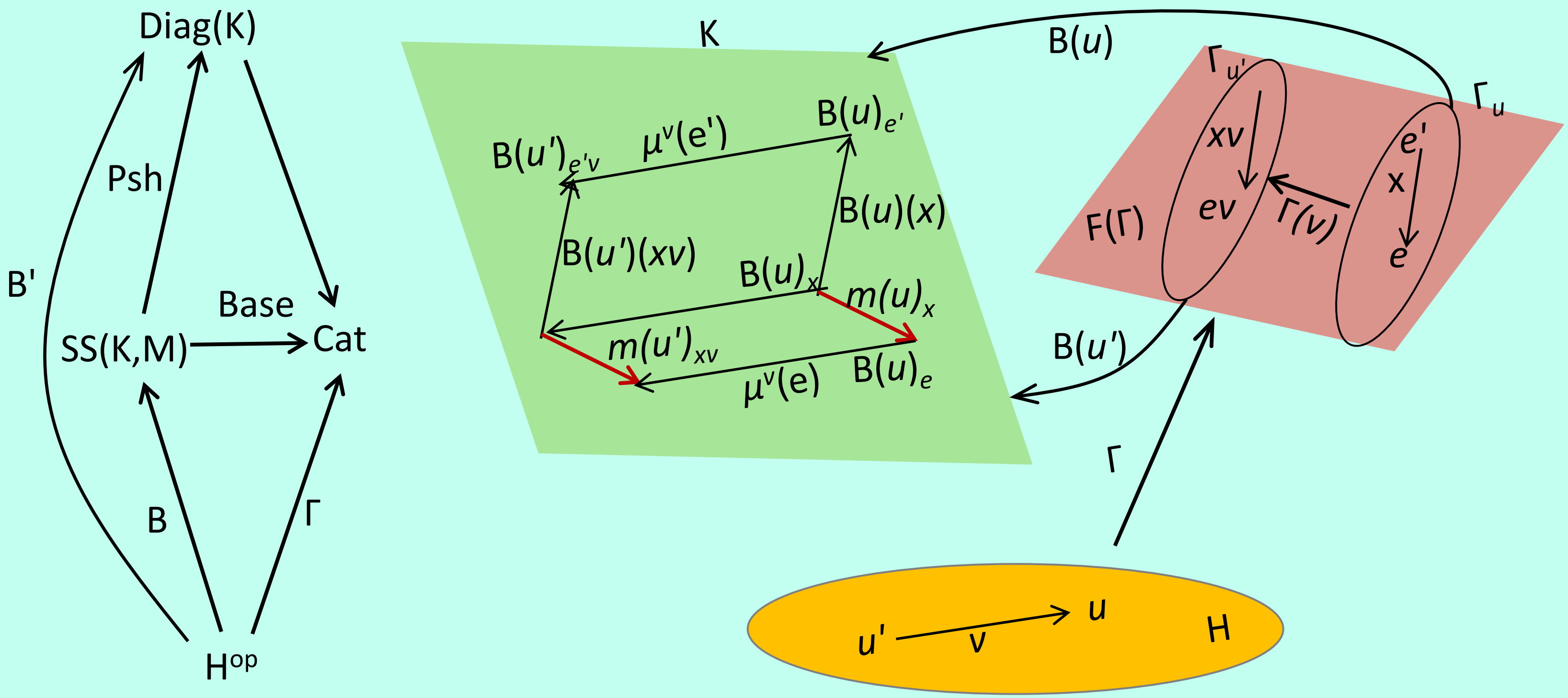
The fiber F'_ε of F' on ε (colimit of G_ε) is the lcs space quotient of the locally convex direct sum of the family of lcs spaces $(F_e)_{y:\varepsilon\to e}$ by the equivalence relation generated by the relations:

$$(y', sx) \sim (xy', s) \text{ where } s \text{ is in } F_x \text{ and } x: e' \to e \text{ in C and } sx = F(x)(s).$$

Theorem (*Associated sheaf*). *If* C *is equipped with a Grothendieck topology* J,, *the category of locally convex semi-sheaves admits a reflection into the category of* Lcs*-sheaves pour* J. *The* J*-sheaf associated to* L *is that associated to* L'.

III. DISTRUCTURES
APPLICATION TO DISTRIBUTIONS

GENERATOR OF (K, M)-**DISTRUCTURES**



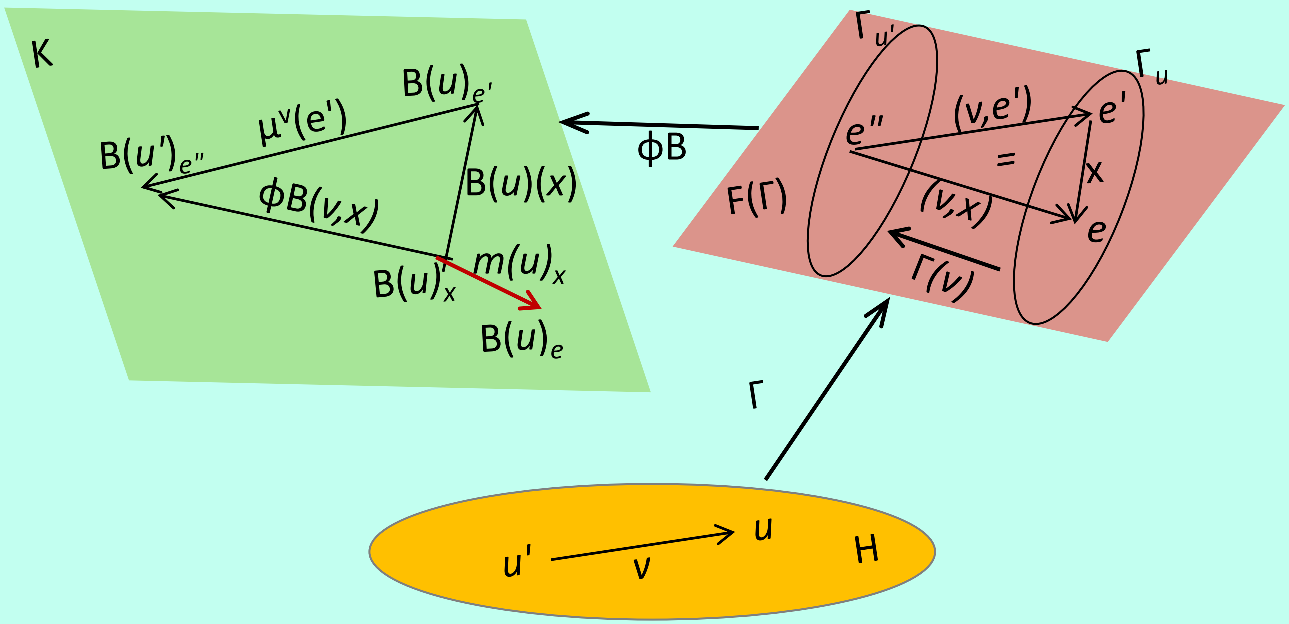
A *generator of* (K,M)-*distructures* on a category H is a functor B from H^{op} to the category SS(K M) of (K, M)-semi-sheaves.

Thus B(u), for an object u of H, is a (K, M)-semi-sheaf on a category Γ_{u}, and B(v) is a morphism ($\Gamma(v)$, μ^{v}): B(u) → B(u'). And the Γ_u are the fibers of a presheaf of categories Γ on H

The composite B' of B with the associated presheaf functor Psh from SS(K, M) to Diag(K) is called the *presheaf of* K-*distructures generated by* B.

Example. If Γ is a constant functor on a category C, then a presheaf of K-distructures identifies with a functor from H^{op} to the category K^{C}, hence with a distributor with values in K.

ANOTHER CHARACTERISATION OF DISTRUCTURES

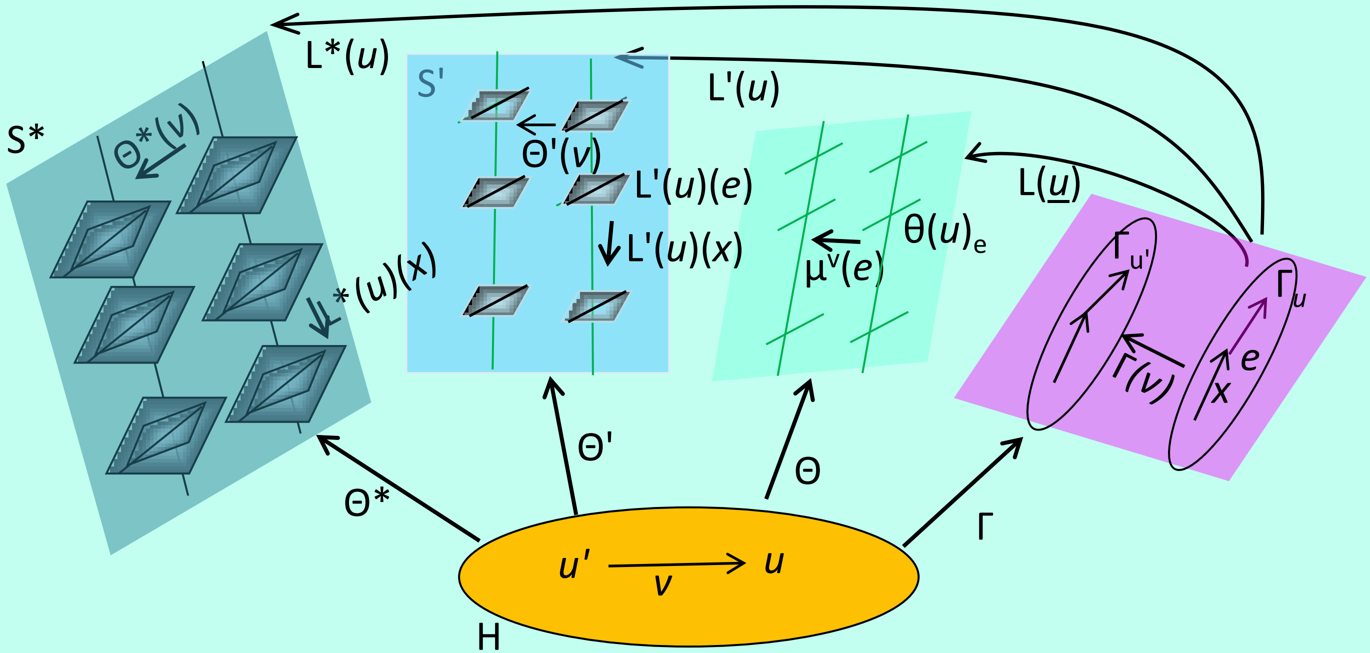


Theorem**.** *There is a* 1-1 *correspondance* ϕ *between the generators* B *of* (K, M)-*distructures with* Γ *as their base* Cat-*presheaf and the* (K, M)-*semi-sheaves on the fibration* F(Γ) *associated to* Γ. *It sends the* Diag(K)-*presheaf* B' *generated by* B *to the* K-*presheaf associated to* ϕB.

The (K, M)-semi-sheaf ϕB on F(Γ) associated to the generator of (K, M)-distructures B admits $B(u)_e$ as its fiber on (*u, e*), and

$$\phi B(v, x) = \mu^v(e')B(u)(x) \text{ for } v: u' \to u \text{ in H and } x : e' \to e \text{ in } \Gamma_u .$$

LOCALLY CONVEX DISTRUCTURES



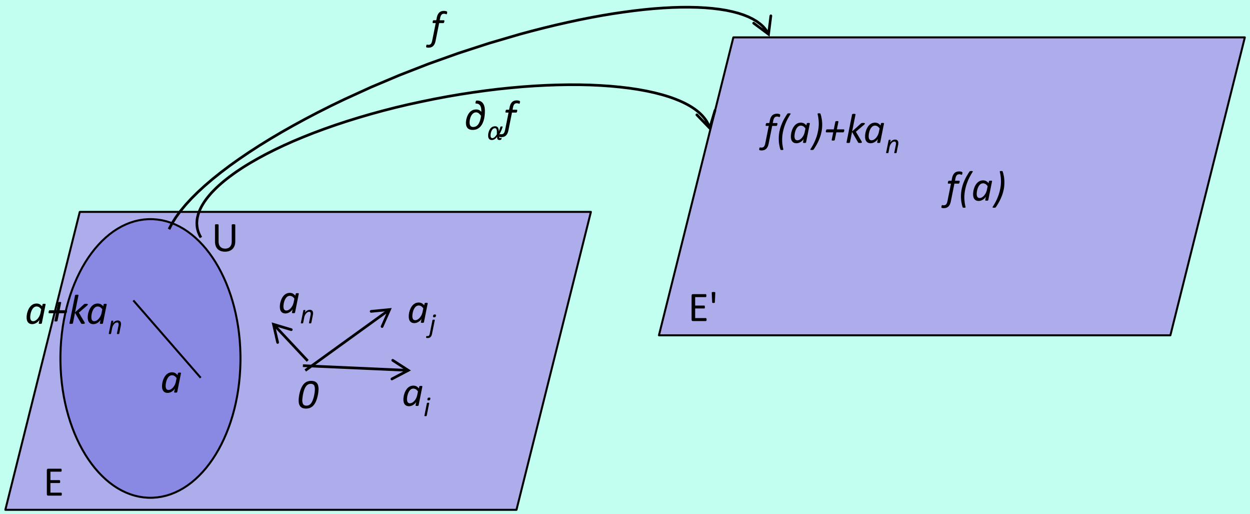
A *generator of locally convex distructures* L, and its generated presheaf of distructures L', are obtained when (K, M) = (Lcs, M_V). Then L and L' have underlying Top-presheaves Θ and Θ', with

$$\Theta_u = \text{ coproduct of the topologies on the fibers of } L(u).$$

The figure emphasizes the 'di-structure' on the topological sum S' of the fibers of Θ': the category H^{op} acts 'horizontally' on it while N^{op} acts 'vertically', where N = coproduct of the Γ_u.

> ***Theorem (Associated sheaf of distructures).*** *If* H *is equipped with a Grothendieck topology* J *and if* Γ *is a sheaf for* J, *then* L *also generates a sheaf* of distructures L* *for* J, *with the same base* Γ, *and whose underlying* Top-*sheaf* Θ* *is the* Top-*sheaf associated to* Θ'.

PARTIAL DERIVATIVES OF A FUNCTION

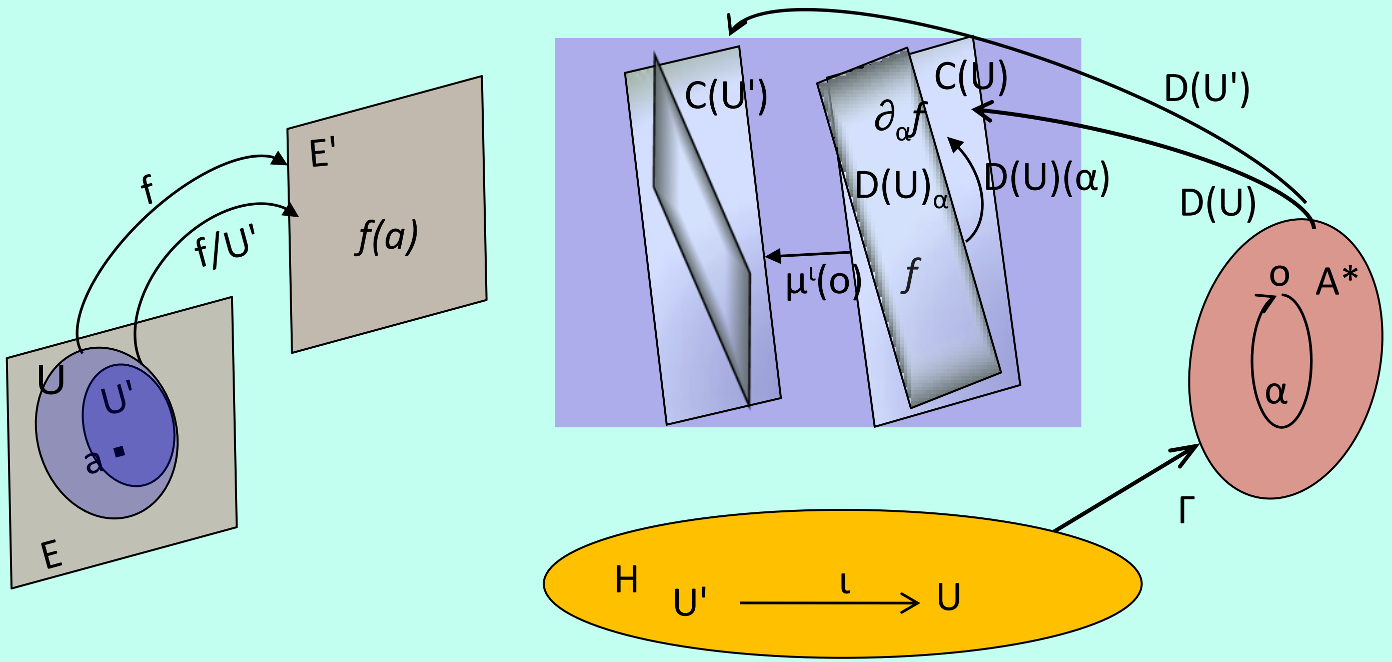


E is a locally convex space and A = $(a_i)_i$ an algebraic base of E. We denote by A* the free commutative monoid on A and by o its unit. An element α of A is a finite multiset $\alpha = \{\alpha_1, \alpha_2, \dots \alpha_n\}$ of *n* (possibly repetitive) elements of A.

Let *f* be a function from an open subset U of E to a locally convex space E'. We say that *f* has an α-*derivative* $\partial_\alpha f(a)$ at *a* if the restriction of *f* to the affine sub-space $a+\Sigma_i \mathbf{R}\alpha_i$ is *n*-differentiable at *a* and admits $\partial_\alpha f(a)$ for its partial derivative with respect to $(\alpha_1, \alpha_2, \dots \alpha_n)$. (It is independent on the order.)

(The following theory of distributions could use another kind of differentiability.)

THE GENERATOR OF E'-DISTRIBUTIONS



E and E' are complete metrizable lcs. The *generator of* E'-*valued distributions on* E is the generator D of locally convex distructures on the category H of open subsets U of E defined as follows:

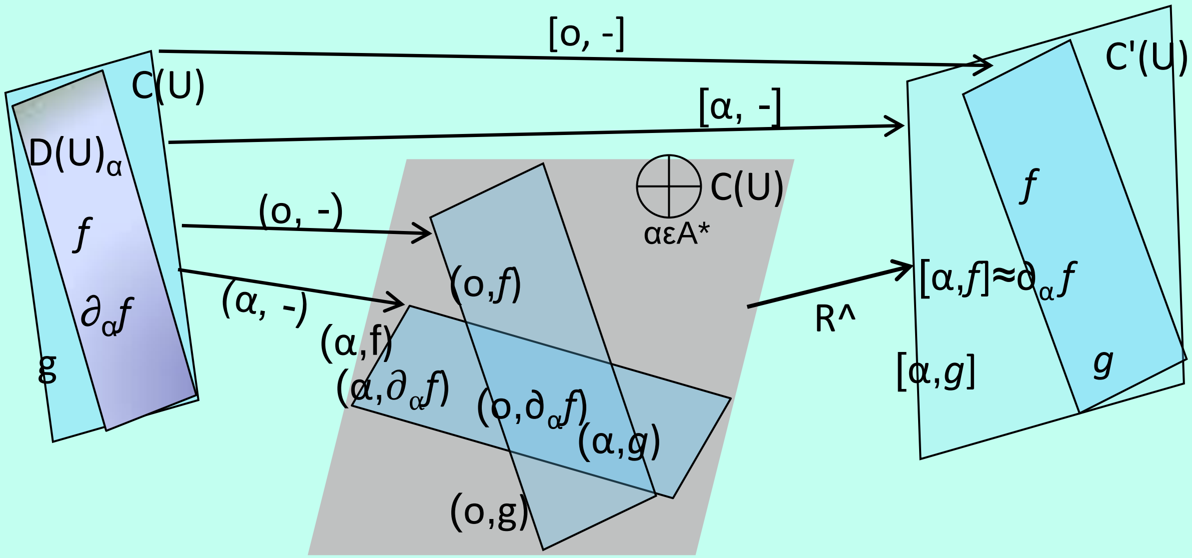
(i) Its base Cat-sheaf Γ is the functor constant on the monoid A*;

(ii) the unique fiber of D(U) is the lcs C(U) of continuous maps from E to E', with the compact-open topology; and, for ι: U' → U, then $\mu^\iota(o)$ is the restriction map *g* |→ *g*/U' from C(U) to C(U').

(iii) $D(U)_\alpha$ is the subspace of C(U) consisting of the functions *f* admitting a continuous α-derivative on U, with the compact-open topology for *f* and all its derivatives up to α, and

$$D(U)(\alpha)(f) = \partial_\alpha f.$$

FINITE ORDER DISTRIBUTIONS



The *presheaf of* E'-*distributions of finite order* is the Lcs$^{A^*}$-presheaf D' generated by D. Its fiber C'(U) at U is the lcs: C'(U) = $\bigoplus_{\alpha\epsilon A^*}$C(U)/R, with R the equivalence generated by:

$$(\alpha\alpha', f) \sim (\alpha', \partial_\alpha f) \quad \text{if } f \text{ is in } D(U)_\alpha.$$

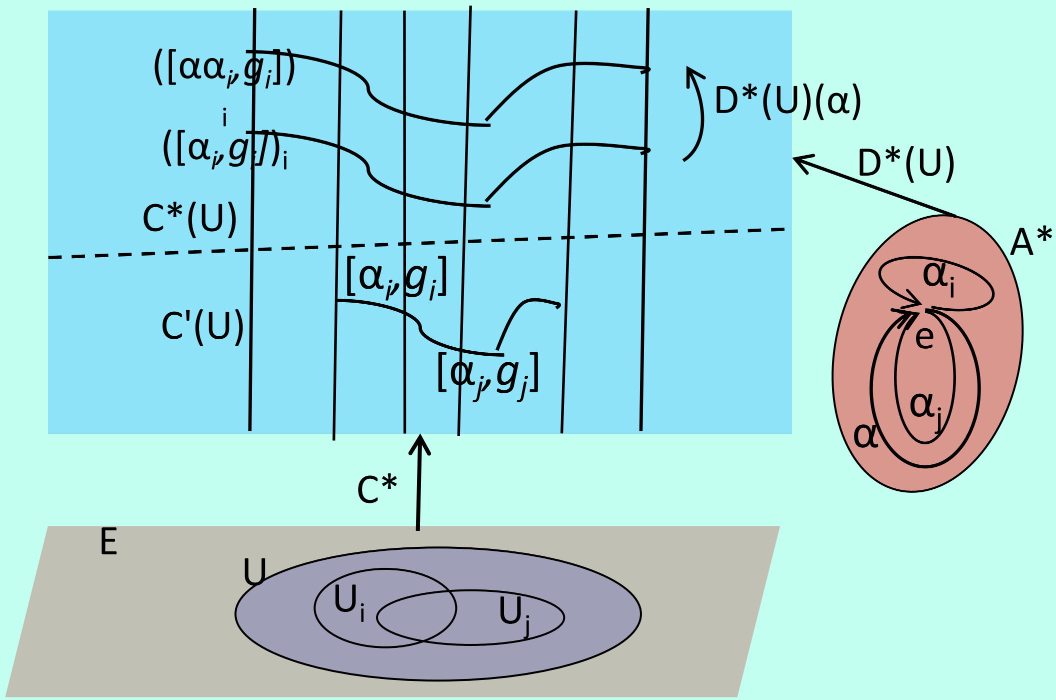
Theorem. *An element of* C'(U) *(or finite order distribution) is an equivalence class* [α, g] *for the equivalence on* A* C(U): (α, *g*) ~ (α', *g'*) *iff there are*

β, β' *in* A* *and g, g' such that* αβ = α'β', $g = \partial_\beta f$, $g' = \partial_{\beta'} f'$ and $\partial_{\alpha\beta}(g - g') = 0$.

The topology on C'(U) *is the final locally convex topology for the maps* [α, -]: C(U) → C'(U): *g* |→ [α, *g*], α ϵ A*. *The map* [o, -] *identifies* C(U) *to a vector subspace of* C'(U). And A* *acts on* C'(U) *via:* α'[α, *g*] = [α'α, *g*].

The proof uses that each continuous function is of the form $\partial_\alpha f$ for some *f* in $D(U)_\alpha$.

THE SHEAF OF DISTRIBUTIONS



The sheaf D* associated to D' is the *sheaf of* E'-*distributions* on E. If U is the union of $(U_i)_i$ an element of its fibre C*(U) is a family $([\alpha_i, g_i])_i$ of elements of C'(U) such that :

$$[\alpha_i, g_i/U_{ij}] = [\alpha_j, g_j/U_{ij}] \text{ on } U_{ij} = U_i \cap U_j.$$

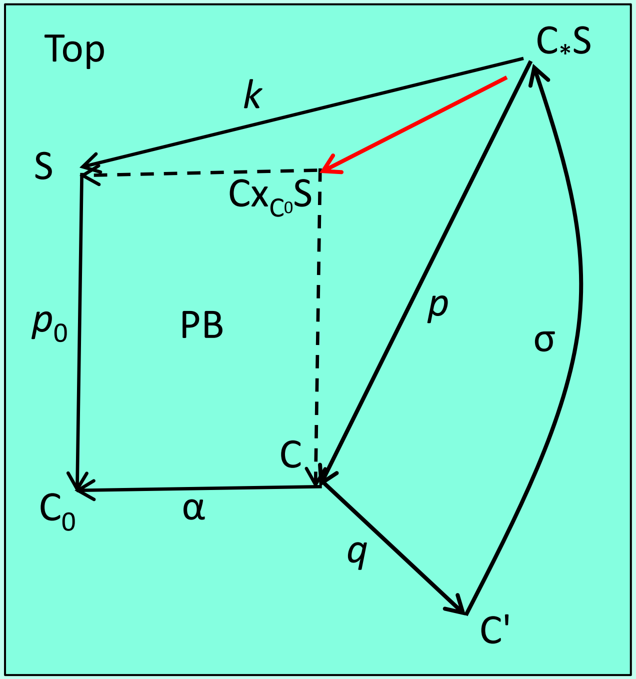
And D*(U)(α) maps $([\alpha_i, g_i]_i)$ on $([\alpha\alpha_i, g_i])_i$.

> ***Theorem.*** *If* E *is a finite dimensional space, the* E'*-distributions coincide with the Schwartz E'-valued distributions.*

If ϕ is a C^∞ function from E to **R** with compact support, the value of [α, *g*] on ϕ is the weak integral

$$[\alpha, g]\phi = (-1)^n \int g \partial_\alpha \phi \quad \text{if } \alpha \text{ has } n \text{ terms.}$$

CONTROL SYSTEMS



A *nucleus of action* is a partial action of a topological category C on the topological space S such that the composition law *k*: (x, s) |→ xs be continuous from an open subset C*S of $Cx_{|C|}S$ to S.

It has an associated 'partial fibration' *p*: C_*S → *C*..

A *control system* is a nucleus of action with a topological functor *q*: C → C'.

A *solution* of this system is a topological functor σ: C' → C_*S section of *qp*.

Differentiable Control Systems are obtained by replacing Top by Diff.

Control Systems provide a categorical frame to model Cauchy boundary problems and variation problems (1964-67)..

The main results give *optimisation theorems* for solutions of a control system which extend the *dynamic programing method* of Bellman. They can be expressed in terms of infinite dimensional vector distributions.

THANKS