



MENS: a categorical model for Emergence and Consciousness

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Online Seminar: "Mathematical Consciousness Science", January 2021

MENS AS A MES BASED ON NEUR

A Memory Evolutive System is a model for bio-socio or cognitive systems, based on a Category Theory framework including Time, It has

(i) a tangled hierarchy of components varying over time, each one 'integrarting' a *pattern* of interacting components of lower levels;

(ii) a multi-agent self-organization by a net of 'local' *Co-Regulators* each with its own temporality and logic..

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MENS is an application of MES to neuro-cognitive systems. The level 0 of its hierarchy, *Neur*, models the neural system. Its higher levels components are called *cat(egory)-neurons*. A cat-neuron of level 1 models a mental object 0 as the 'binding' (or colimit) of a synaptic pattern activated by O. A *cat-neuron* of level n+1, is obtained by iterative binding of patterns of lower level cat-neurons which model flexible mental objects or processes of increasing complexity. Whence an '*algebra of mental objects*' (in the sense of Changeux).

The number of levels increases over time, allowing for the formation of a robust and flexible Memory with a higher level, called *Archetypal Core*, acting as a motor in the formation of conscious processes..

GRAPHS AND CATEGORIES



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Path of the graph = sequence of consecutive arrows.

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The paths of a graph form a category (composition = convolution).

A *functor* F: C \rightarrow K is a graph homomorphism which respects the identities and the composition. A *partial functor*: F': C \rightarrow . K is a functor from a full subcategory C' of C to K..



SHEAF AND ES OF NEURONS



Let T be the interval of **R** which represents the timeline of the neural system of a person.



Definition: For an interval I of T, let Neur, be the neural category on I so defined: an object N₁ models the restriction at I of a neuron N whose support contains I; a morphism $f_i: N_i \rightarrow N'_i$ models a synaptic path from N_i to N'₁. For I = {t}, *Neur*₁ is denoted *Neur*₄

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Proposition. (i) The categories *Neur*₁ where I is an open interval of T, define a sheaf of categories *Neur* on (the topology of) T.

(ii) For t < t' there is a partial functor **Neur**(t, t'): $Neur_t \rightarrow Neur_{t'}$ called *transition from t* to t', mapping the state N_t of N at t to its state N_{t'} if N still exists at t'. These functors **N**(t, t') define the *Evolutive system of neurons* **NEUR** which will be a sub-ES of the ES **MENS**.

EVOLUTIVE SYSTEM [EV 1987]



Evolutive System K:

It consists of:

(i)an interval T of R;

(ii) For each t ϵ T, a *configur-ation* category K_t ;

(iii) For t < t', a transition partial functor $\mathbf{K}(t,t')$: from (a subcategory $K_{tt'}$ of) K_t to $K_{t'}$; these data defining a functor

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Component N of K = maximal family of objects N_r of K_t related by transitions. *Links* between components are defined similarly.

For I =]t, t'[, let K_1 be the category whose objects N₁ are restrictions to I of the components N whose support contains I; the morphisms are similarly defined from links. These categories form a *sheaf K of categories* over T, satisfying; (A) For each $s \in I$ the functor from K_1 to the stalk K_s is injective.

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Proposition. Every sheaf K satisfying (A) is associated to an ES. K

NEURAL ACTIVITY PATTERNS AT THE BASIS OF MENS



From: "The Brain from top to bottom", McGill

In 1949 Hebb emphasized the role of *cell-assemblies:* "Any frequently repeated, particular stimulation will lead to the slow development of a "cell-assembly" <...> capable of acting briefly as a close system".

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Now the role of such cell assemblies, or synaptic patterns, or activity patterns, or neuronal groups (Edelman) is recognized as essential for the formation of mental objects. The pattern will be modeled by a diagram P in a category *Neur*₁. The mental object it represents, when integrated, will be represented by the *colimit* cP of P in *Neur*₁ if it exists, and otherwise by the colimit cP of P in the larger category *MENS*₁; in this last case, cP is called a *cat(egory)-neuron* (of level 1)..

COLIMIT OF A PATTERN

A pattern (or diagram, or just a subgraph) P in a category K consists in a family of objects P_i and some distinguished morphisms $f: P_i \rightarrow P_j$

A cone from P to A is a family of links s_i : P_i \rightarrow A correlated by the distinguished morphisms of P so that $s_i f = s_i$



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Definition. A pattern P admits a *colimit* (or *binding*) cP in the category K if there is a cone (c_i) from P to cP through which any cone (s_i) from P to any A factors uniquely through a morphism

s: cP \rightarrow A, meaning that s $c_i = s_i$ for each *i*.

Roughly, the colimit cP of P 'integrates' the information contained in P

BINDING LINKS. COMPARISON SUM \rightarrow COLIMIT

Given 2 patterns Q and P in K a *cluster* G from Q to P is a maximal set of morphisms from each Q_k to a zig-zag of P_i 's, correlated by morphisms in Q.

Proposition. If Q and P have colimits cQ and cP, a cluster G from Q to P 'binds' into a unique g: cQ -> cP, called a (Q, P)-*binding link.*

Whence a solution to the Binding Problem.



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Aristotle

Important case: $Q = \{P_i\}$ Then cQ is the *sum* of P and $\{id_p\}$ generate a cluster from Q to P which binds into a *comparison morphism*

c: sum(P) \rightarrow colimit(P)

which measures the 'lack of coherence'' of P " the degree to which a system is "more than the sum of its parts'. Koch equates **Phi** to such a lack of synergy

MULTIPLICITY PRINCIPLE (MP) MODELING DEGENERACY

Edelman introduces the *degeneracy of the neural code*: "More than one combination of neuronal groups can yield a particular output, and a given single group can participate in more than one kind of signaling function." (The remembered present, 1989, p. 50)



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Definition. 2 patterns P and P' of K satisfy the MP is they have the same colimit C and there is no cluster between P and P' bnding into the identity of C. Then C is called a multifaceted object of K.

Degeneracy in **NEUR** means that some cat-neurons are multifaceted components of **MENS**

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Example; K_0 is the subcategory *Neur*₁ of the category $K = Mens_1$ and C is a multifaceted cat-neuron (of level 1).

The multipart link gg': $B \rightarrow C'$ '*emerges' in K* though not 'physically' observable via morphisms connecting objects of Q' and P'. In fact it depends on the 'global' structure of K which 'imposes' that Q and P have the same colimit.

The passage from K₀ to K is an example of the Complexification Process



A *di-sketch* d = (**D**, Γ) on a category K_0 consists of a set **D** of patterns without a colimit in K_0 and a set Γ of cones (colimit-cones or not).

d-Complexification Process: Find a functor X: $K_0 \rightarrow K$ to a category K, called the d *complexification* of K_0 such that X(Q'), for each Q' \in **D**, acquires a colimit cQ' in K and each $\gamma \in \Gamma$ is mapped on a colimit cone



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Complexification Theorem. Given a di-sketch d on K_0 , there exists a universal solution to the d-Complexification Process If K_0 satisfies MP, so does K and there is weak emergence of colimit-cones and 'global' emergence of multipart links in K (with impredictibility for an external observer of Brian Johnson).

An explicit construction of *K* (by induction) is given in [EV, 1987].

GENERAL STRUCTURE OF MENS



MENS is a *Hierarchical ES* (i.e. an ES whose configuration categories are hierarchical). It admits **NEUR** as its sub-ES of level 0.

For an interval I of T, the hierarchy of the category $Mens_1$ is constructed, from Neur₁ up, by a sequence of complexifications. Each of them leads to the formation of catneurons of level n+1 obtained as colimis of patterns of cat-neurons of levels \leq n.

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A cat-neuron of level n+1 gives a 'dynamic' model of a mental object O by becoming the colimit cP = cP' in $Mens_1$ of the various patterns pattern P, P' of (cat-)neurons of lower levels able to activate O.

Each complexification leads to the *synchronous emergence* of multifaceted cat-neurons of level n+1 and of multipart links connecting them.

HIERARCHICAL CATEGORIES

Definition. A category is hierarchical if its objects are distributed into different complexity levels (from 0 to m) so that an object C of level n+1 is the colimit of at least one pattern of connected objects of levels \leq .n.



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In MENS. A cat-neuron C of level n+1 has such a ramification whose neural base represents a *physical (neuronal) realizability of C*. Due to MP, a *multifaceted cat-neuron* C *has multiple physical realizabilities* which emerge in a dynamic way (through the gradual unfolding of a ramification). Thus there is no isomorphism between physical and mental states. This makes possible the development of a *neurophenomenology* in the sense of Varela as shown in [EG, 2015].

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If each object of *K* is of complexity order 0 or 1, we say that *K* resorts to a *pure reductionism*.

Emergentist reductionism (Mario Bunge). MP is necessary for the existence of objects of complexity order > 1 and may lead to the emergence of objects of increasing orders via complexifications.

=> **MENS** resorts to such an emergentist reductionism.





DYNAMIC OF MENS. FORMATION OF A MEMORY

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Memory. An 'external stimulus' O may activate a pattern P of neurons of levels \leq n at an instant *t*; if the activation subsists long enough, it will lead to the formation of a cat-neuron cP colimit of P via a complexification from *t* to a later *t'*, depending from the propagation delays of the morphisms of P. We call cP a *record* of O. Such a record can be later 'recalled' via the unfolding of one ramification down to its neural base.

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MENS develops a robust though flexible hierarchical Memory **Mem** with multifaceted records of increasing complexity levels.

MENS AS A MULTI-AGENTS SYSTEM

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Each CR has a hybrid dynamic: It acts stepwise, at its own rhythm and with a differential access to **Mem**, e.g. to recall the *admissible procedures* Pr corresponding to its function. During each step it forms its actual *landscape* consisting of the active links which process information to CR, selects an admissible procedure and sends its commands to effectors E. (Such a step can be modelled by PDE.)



Landscape of CR = ES whose components are the curved arrows

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The commands sent by different CRs at a given time *t* can be conflicting, and must be harmonized, with risk of a fracture for a CR whose commands are not, effected.

STUCTURAL CORE OF THE BRAIN



In 2008, Hagman & al. discovered the

" existence of a structural core in human cerebral cortex [...] both spatially and topologically central. [...] an important structural basis for shaping large-scale brain dynamics "<...> " linked to self-referential processing and consciousness."

This core is a central subgraph of the graph of neurons which forms a "rich club", with many strongly connected hubs.

ARCHETYPAL CORE AC



Development over time of the **Archetypal Core AC**, a higher sub-ES of the memory **Mem**, based on the Structural Core. Its components are multifaceted records of higher levels with many ramifications, which integrate significant memories of different modalities, often recalled.

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Intentional CRs. Higher co-regulators CR_i based on Crick's "conscious units" (in the associative cortex) and directly connected to AC constitute, with the binding or multipart links between them, a *cs-agent network* acting as a macro-co-regulator.















RETROSPECTION IN THE MACRO-LANDSCAPE

ML has a longer timeline than the landscapes of the CR_i since **AC** acts as a motor in its activation. It develops using the following processes:

(i) Propagation of the activation through loops and unfolding of ramifications;(ii) Sharing of information by the CR;

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ML acts as a 'mental space' in which is operated a *retrospection process* integrating processes of different temporality and complexity. Some are conscious processes; others are 'non-conscious' (e.g. instinctive or perceptive behaviours, emotions and affects, reflexes), thus allowing **ML** to support *embodied cognition*; still others relate to *phenomenal data*.

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The retrospection and prospection processes are to be compared to Husserl :

"Il y a dans le présent une *rétention* du passé (rétention primaire si c'est un passé immédiat, rétention secondaire si c'est un souvenir plus lointain) et une *protention* du futur (de ce qui va immédiatement arriver). "



CONSCIOUS PROCESSES



An unexpected or striking event S activates part of AC

===> formation of a long term macrolandscape **ML**, in which *conscious processes* develop by integration of time via:

1. *Retrospection:* analysis of the situation and recall of near past for "making sense" of S (by abduction).

2. **Prospection**: search of "scenarios" by iteratively constructing virtual landscapes ("mental spaces") in **ML** in which sequences of procedures are tried, by evaluation of the corresponding complexifications.

CONCLUSION

| MIND | MENS | BRAIN |
|------------------------------|---|--|
| Simple Mental Object | (Cat)-neuron level 0 Cat-neuron of level 1 | Neuron Class of (equivalent) neuronal patterns |
| Algebra of mental objects | Cat-neuron of level n | Class of patterns of neuronal |
| Emerging Properties | Multifaceted objects Multipart links | Degeneracy of the neural code |
| Memories Self | Memory Archetypal Core | Structural core of the brain |
| Conscious processes | Macro-Landscape Retro- and prospection | Structural core of the brain Consciousness loop (Edelman) |

MENS proposes a theory of mind, in which a hierarchy of mental objects and processes emerges from the functioning of the brain. We show how *the degeneracy property* (= MP) is the characteristic making this emergence possible, thus allowing the development of the Archetypal Core, which act as a motor in the formation, of higher cognitive processes up to consciousness.

FOR MORE INFORMATION

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The following internet sites contain a large number of our papers:

https://ehres.pagesperso-orange.fr https://vbm-ehr.pagesperso-orange.fr

THANKS