CATEGORIES IN COGNITION:

AN INTEGRATIVE APPROACH TO MULTI^3-SYSTEMS

by

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Cognition can develop in biological, socio-economic or even artificial systems which are:

- ¤ evolutionary,
- ¤ self-organized,
- ¤ multi^3-systems, i.e. multi-level, multi-agent, multi-temporality.

Most models retain only some of these properties

Problem. How to use Category Theory for constructing an integrative approach?

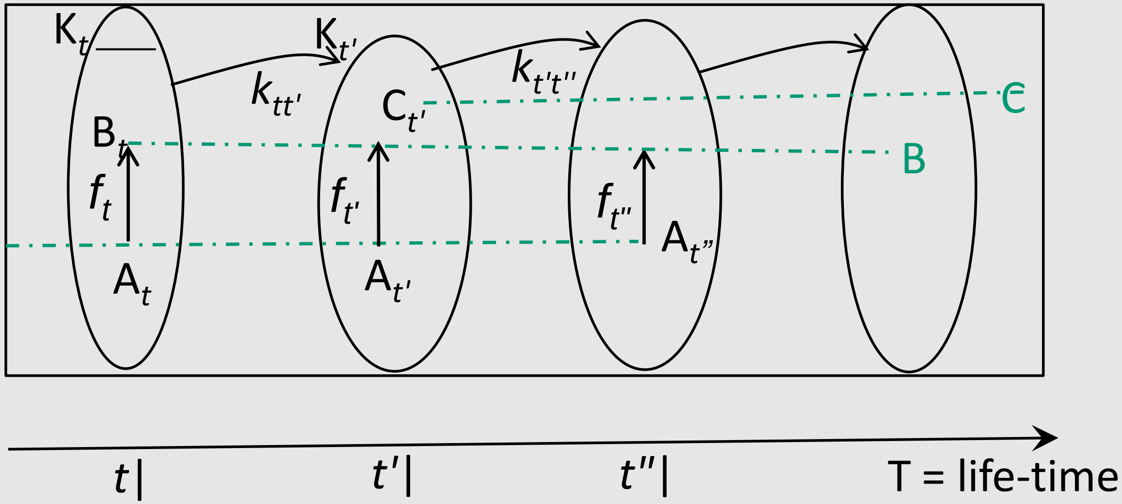
Memory Evolutive Systems (MES, Ehresmann & Vanbremeersch, 2007) are a step in this direction. They account for the above properties, studying the system at work and its 'becomng' from the inside.

But they raise difficult computational problems.

Part I

HIERARCHICAL EVOLUTIVE SYSTEMS

EVOLUTIVE SYSTEM



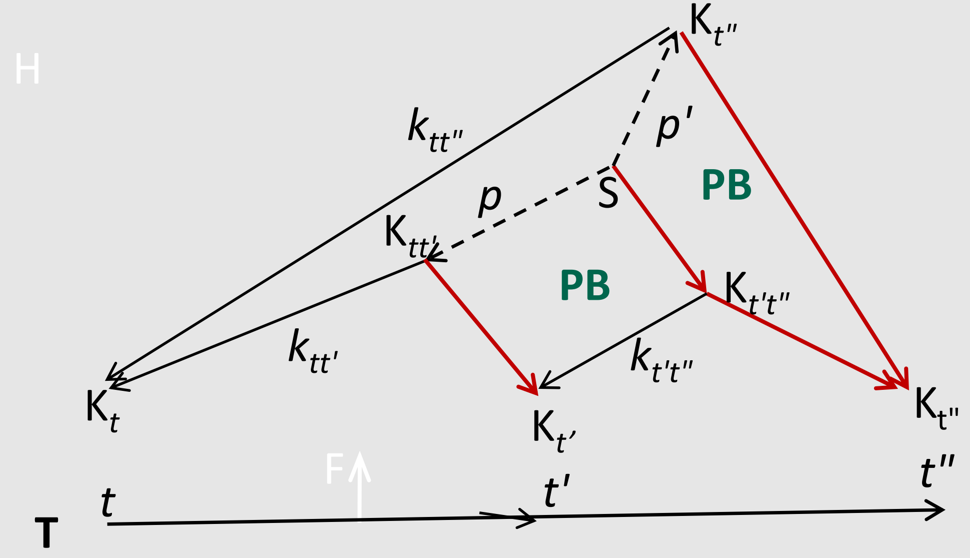
Configuration of the system at a time t = category K_t: its objects represent the state A_t of the components A existing at t, morphisms, channels through which they may interact.

***Evolutive System* K** : it consists of a family of configuration categories (K_t) indexed by an interval T of **R** (= 'life' of **K**) and a family of partial '*transition*' functors $(k_{tt'}: K_{tt'} \to K_{t'})_{t<t'}$ satisfying the condition: If $k_{tt'}(A_t) = A_{t'}$ if defined, then $k_{tt''}(A_t)$ is defined if and only if $k_{t't''}(A_t)$ is defined.

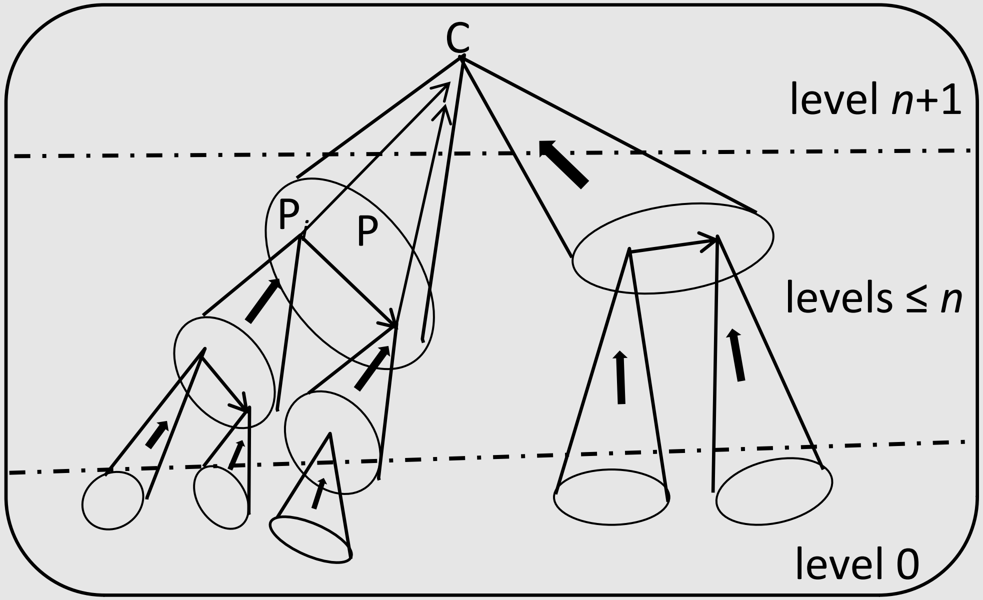
A *component* A of **K** is a maximal family (A_t) of objects related by transitions; and similarly a *link* is a maximal family (ft) of morphisms. Components and links defined on all an interval J of T form a category K_J

Example: the *Evolutive System of Neurons* **Neur**: its configuration Neur_t at t is the category of paths of the graph whose objects represent the neurons existing at t, and the arrows the synapses between them. The components of **Neur** are the neurons and the links the synaptic paths.

An evolutive system **K** can also be seen as a 'partial'-fibration over the category **T** defining the order on the interval T of **R**. Formally:
K can be defined as the lax-functor from **T** to the 2-category of partial functors, satisfying a supplementary 'pullback' condition, as indicated in the figure (red arrows are insertions in *Cat*).

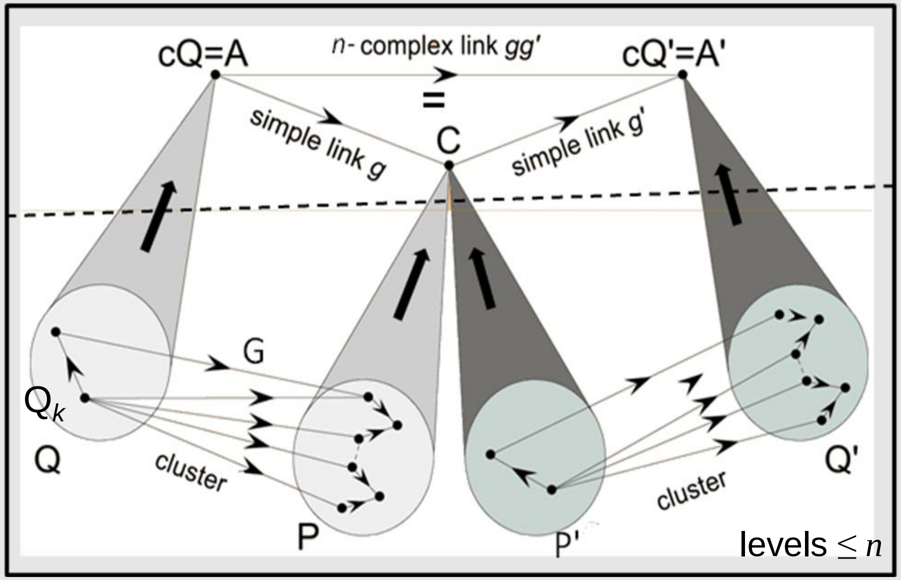


HIERARCHICAL CATEGORY



A ***hierarchical category*** is a category K in which each object is attributed an integer (between 0 and m), called its *level* so that the following property is satisfied: an object C of level n+1 is the colimit of at least one diagram P in K (taking its values P_i) in the levels ≤ n; P is called a *decomposition* of C.

An object C of a hierarchical category K has at least one *ramification* down to level 0. The *complexity order* of C is defined as the shortest length of its ramifications.

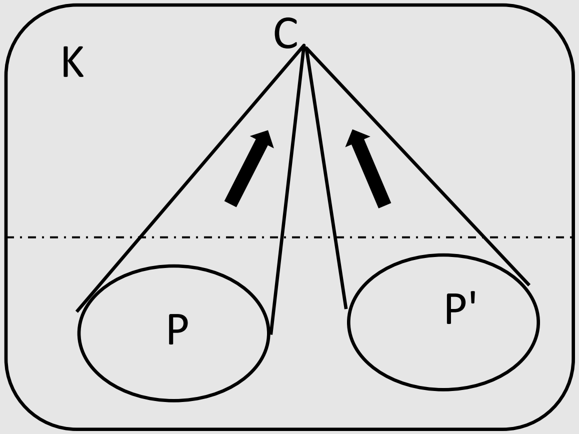


If Q and P are two diagrams in K, a ***cluster*** from Q to P is a maximal set G of morphisms such that: (i) For each Q_k there is at least one f: $Q_k \rightarrow P_j$ in G; and if there are several such links, they are correlated by a zigzag of morphisms of P. (ii) The composite of a morphism in G with a morphism of P (on the right) or of Q (on the left) belongs to G.

If P and Q have colimits cP and cQ in K, the cluster G 'binds' into a unique morphism g: cQ → cP called a (Q, P)-*simple link*, or just an ***n*-simple link** if the objects in P and Q are of level ≤ n.

An ***n*-complex link** is a composite of n-simple links which is not n-simple.

MP. COMPLEXITY THEOREM



Two diagrams P and P' in K are *weakly homologous* if they may have the same colimit C though there is no cluster between them binding into an Identity of C. (This property has been introduced to generalize the *degeneracy* property of the neural code (Edelman, 1989).)

Definitions. (i) A component C of a hierarchical category K is *n-multifaceted* if it is the colimit of at least two diagrams in the levels ≤ *n* which are weakly homologous. (ii) 2. K *satisfies the **Multiplicity Principle*** (MP) if it has *n*-multifaceted objects.

Complexity Theorem. *In a hierarchical category, the Multiplicity Principle is a necessary condition for the existence of objects of complexity order* > 1 *and of n-complex links*.

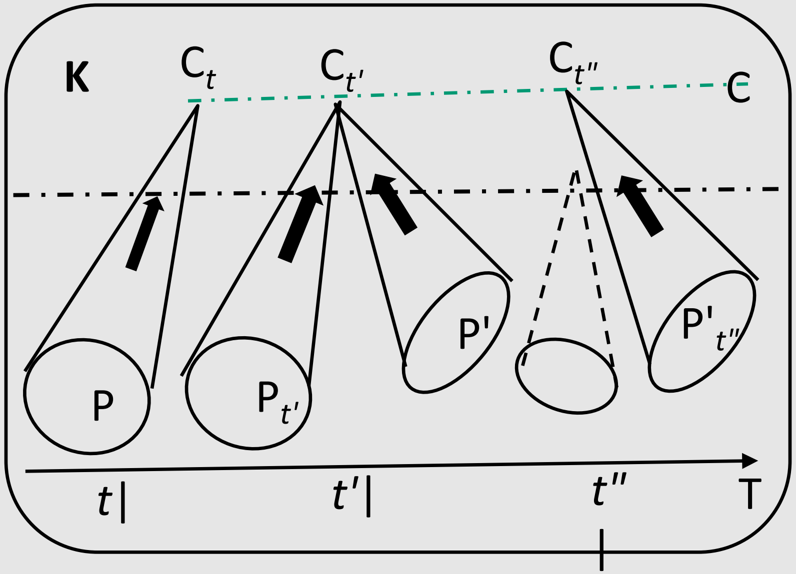
Problem. How to account for MP in computations?

HIERARCHICAL EVOLUTIVE SYSTEM

Definition. A ***hierarchical Evolutive System*** (HES) is an Evolutive system K such that: the categories K_t are hierarchical and satisfy the Multiplicity Principle, and the transitions respect the levels.

MP gives flexibility to the system:

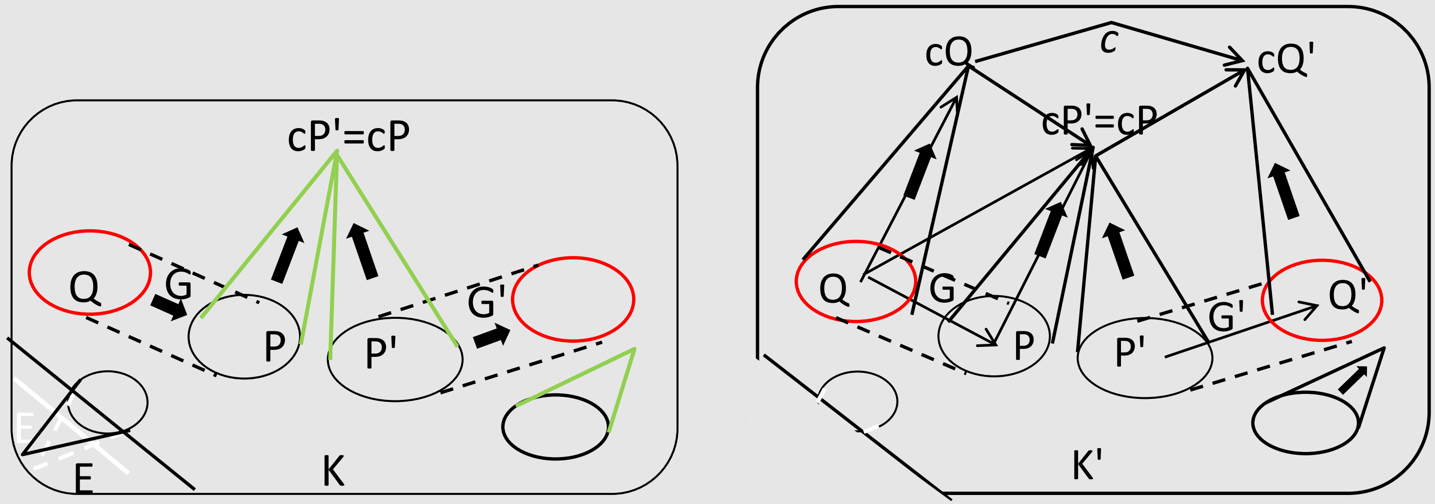
A multifaceted component C is adaptive: at a given time it can operate through any of its decompositions P, and switch between them. Over time it can take its own individuation, independent from its first decomposition P.



Part II

COMPLEXIFICATION. EMERGENCE

COMPLEXIFICATION PROCESS



Definition. A *pro-sketch* (or *procedure*) consists of data S = (K, E, I, D), where K = multiplicative graph, E = subset E of K, I = a set of inductive cones (in green), D = a set of diagrams (in red). A *model* of S in a category N is a partial functor F from K to N satisfying: it is defined on the largest sub-category of K not including E, it transforms the cones in I into colimit-cones in N, and FQ for each Q in D, acquires a colimit cQ in N.

Complexification Theorem. *A pro-sketch* S *on a category* K *has a smallest model* K', called the ***complexification** of* K for S.

K' is explicitly constructed as the prototype of the sketch generated by S.

Iterated complexification Theorem. *If a category* K *satisfies* MP, *so does a complexification, and two successive complexifications of* K *are not reducible to a unique complexification for an adequate pro-sketch on* K.

Emergence Theorem. *In a HES* **K** *the transitions are obtained by composition of complexifications which can lead to the emergence of components of increasing complexity order.*

The emergence of more and more complex components allows cognition by development of an evolutive sub-system **Mem** of **K** called the ***memory*** storing items of any kind.
A multi-faceted component C in **Mem** is a robust though flexible memory: C can be recalled under any of its multiple decompositions to recognize the item it memorizes under different forms, and it adapts to changing situations by acquiring new decompositions and suppressing those which are no more valid.

Part III

SELF-ORGANIZATION: MULTI-AGENT DYNAMICS

MEMORY EVOLUTIVE SYSTEM

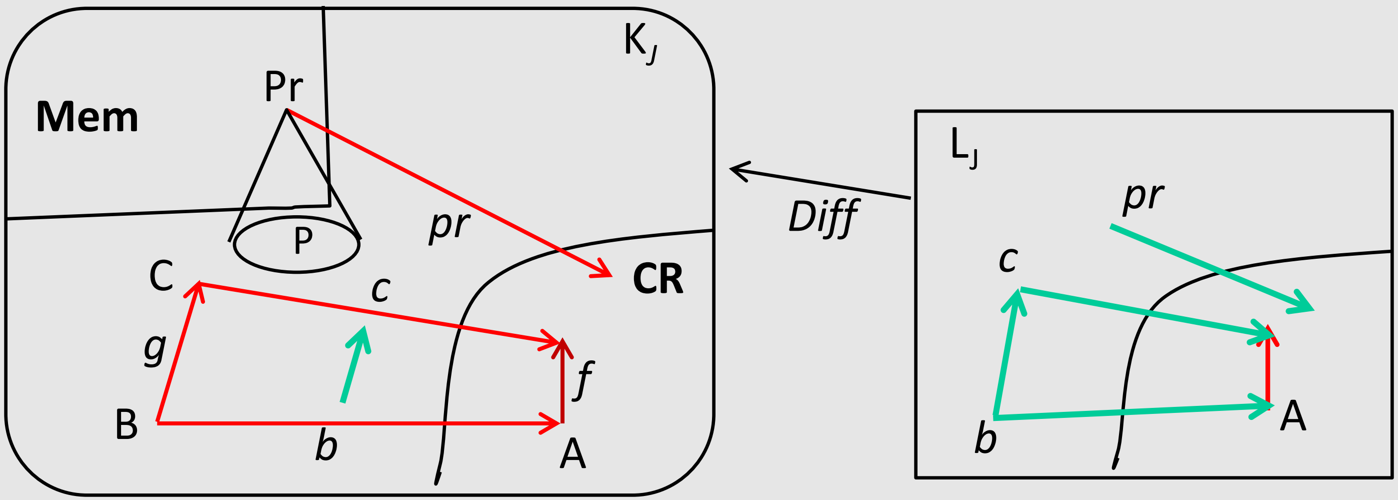
A ***Memory Evolutive System*** (MES) is defined by: a hierarchical evolutive system **K**, a sub-system **Mem** of **K** called the *Memory*, and a heterarchical net of specialized sub-systems called *co-regulators* (CRs); each **CR** acts stepwise at its own rhythm and has a differential access to **Mem**, in particular to store and retrieve the 'actions' it can perform, memorized by the set of its CR-*admissible actions* (in a sub-system **Proc** of **Mem**).

Temporal constraints:

(i) A link between components has a *propagation delay* at each *t* where it exists; these delays (representing the time of transmission of interaction) are 'measured' by a functor from K_t to the additive monoid $\mathbf{R}_+$.
The intermittence of transmission is accounted for by a functor from K_t to the multiplicative monoid {0, 1} associating 0 or 1 to the link depending if it is 'passive' or 'active' at *t.*

(ii) The dynamic of a co-regulator is “hybrid’, i.e. regulated by 2 different timescales: its own discrete timescale delimiting its successive steps, and the continuous timescale T which supports its operations during each step.

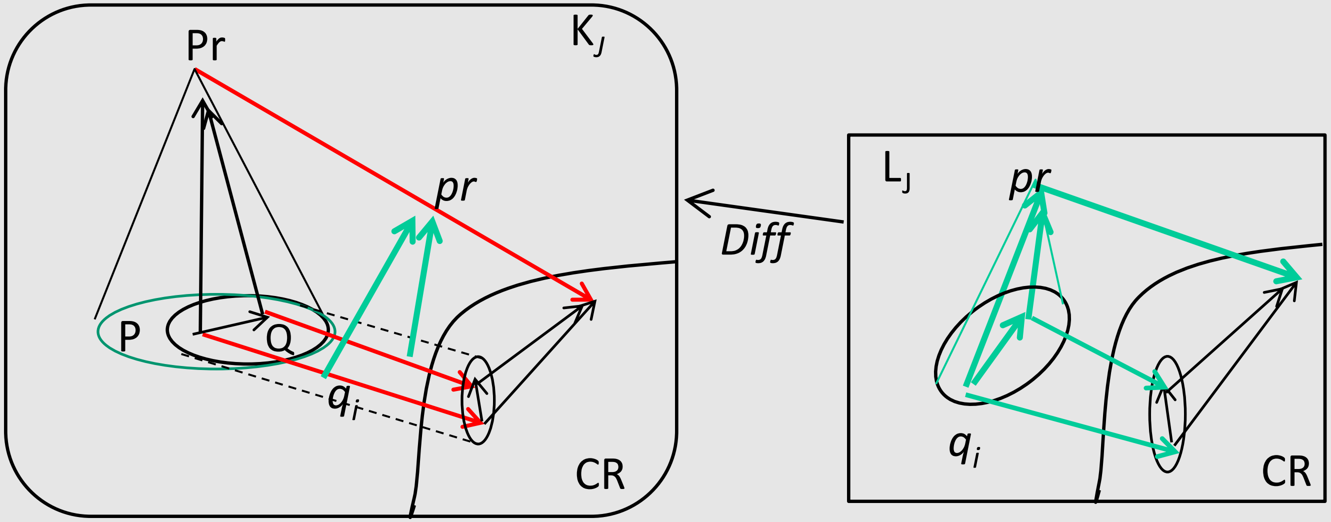
OPERATING DYNAMIC OF A CO-REGULATEUR CR



One step of the co-regulator **CR** extends between two consecutive instants of its discrete timescale, say during the interval J = [*t, t'*[.

1. The (inside) *view* of **K** for **CR** is modeled by the full sub-category L_J of the comma-category $K_J|CR_J$ whose objects are the links *b* from B to a component A of CR active on J. L_J is called the ***landscape of*** CR on J; it is equipped with a functor *diff(erence)* associating B to *b.* We identify CR_J to a sub-category of L_J

Remark. L_J is the split fibration associated to the presheaf of categories on K_J associating to a component B of K_J the sub-category of B|CR having for objects the links active on J (cp. to the *agent view* defined by Golas in MAS).



2. Selection of an *action.* A CR-admissible action Pr is selected in L_J via *pr* and it is realized through one decomposition P of Pr.
To P we associate a pro-sketch S on K_J such that:

update of **K** at t' after the action = complexification K' of K_t for S
=> 'Real' update of the CR view at t' = next landscape L' of CR in K'.

However P might be viewed by CR only through a sub-diagram Q of P, whence associated to a pro-sketch S' on L_J

=> CR-expected update of L_J at t' = complexification L" of L_J for S'.

L' and L" can be different => fracture for CR.

GLOBAL DYNAMIC OF THE SYSTEM

The global dynamic of the system at a given time *t* should take account of the different local dynamics of the co-regulators at t. Formally let us denote by S_i the pro-sketch selected by the co-regulator CR_i around *t.* The pro-sketch considered to update K_t should be

$$S = (K_t, U_iE_i, UI_i, UD_i).$$

However this pro-sketch may not exist, e.g. if a diagram in D_i is partially in E_j for another *j*. Thus there is need of an equilibration process, called the *interplay among the CRs*, to find a 'best' compromise.

MP gives flexibility to this interplay: S_i is associated to a particular decomposition P_i of Pr_i; a switch between P_i and another decomposition P'_i of Pr_i could lead to a pro-sketch S'_i causing less incompatibilities.

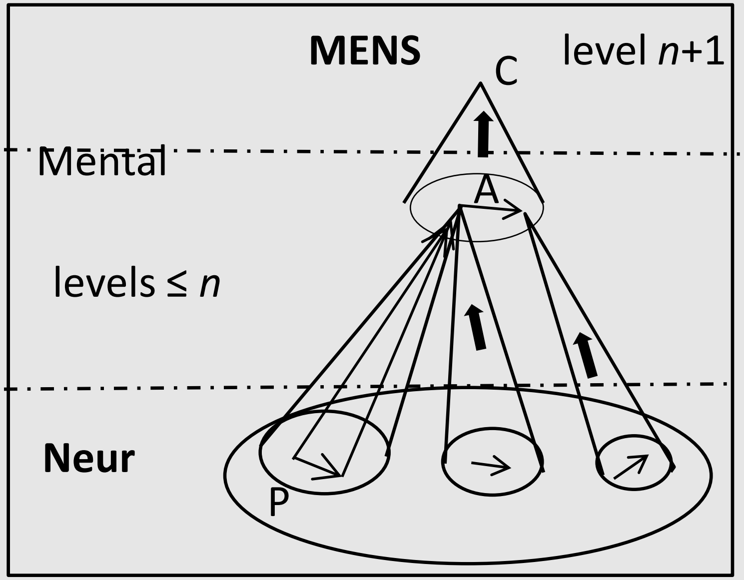
The problem is still complicated by the different rhythms of the CRs. One step of CR_i can cover several steps of CR_j , and changes for CR_j can occur too quickly for being viewed in 'real time' by CR_i. This is the problem of the *dialectics between heterogeneous co-régulateurs*.

Problem. How to compute the interplay among CRs?

Part IV

APPLICATIONS TO COGNITION. MENS

MEMORY EVOLUTIVE NEURAL SYSTEM/ MENS



At the basis of **MENS**:
A mental object is characterized by each of the synchronous assemblies of neurons it can activate (Hebb, completed by Edelman's degeneracy of the neural code).

Starting from the above fact, we have constructed an integrative model of the human neuro-cognitive-mental system, **MENS.** It is obtaned by successive complexifications of the evolutive system of neurons **Neur**: it has **Neur** as its level 0**,** while its higher level components, called *category-neurons*, model more and more complex mental objects constructed as iterated colimits of (classes of weakly homologous) synchronous assemblies of neurons. A category-neuron has multiple 'physical' realizabilities through the unfolding of a ramification down to the neuronal level.

ARCHETYPAL CORE. HIGHER COGNITION

MENS develops a flexible internal model, the ***Archetypal Core.*** **AC** is a sub-system of **Mem** obtained by successive complexifications of the cortex structural core (discovered by Hagman & al., 2008); its higher order components are connected by loops of fast complex links (forming a Grothendieck topology) self-maintaining their activation. We have shown how it allows the formation of a longer term *macro-landscape* M, which plays an essential role in the development of higher cognitive processes, e.g. consciousness, anticipation or creativity.

Applications in "futures studies" and in "innovative design" (with M. Béjean) show that the formation of a collective 'archetypal core' is essential for creative prospective and innovation.

Remark. Applications of MES have also been given in biology, e.g. an aging theory for an organism (with J.-P. Vanbremeersch) or the WLIMES model developed with P. Simeonov.

FOR MORE INFORMATION

Memory Evolutive Systems: Hierarchy, Emergence, Cognition, Elsevier, 2007.

MENS, a mathematical model for cognitive systems, *JMT* 0-2, 2009.

MENS, an info-computational model for (neuro-)cognitive systems up to creativity, *Entropy* 14, 2012, 1703-1716.

A theoretical frame for Future Studies, On *the Horizon,* Volume 21 issue 1, 2013.

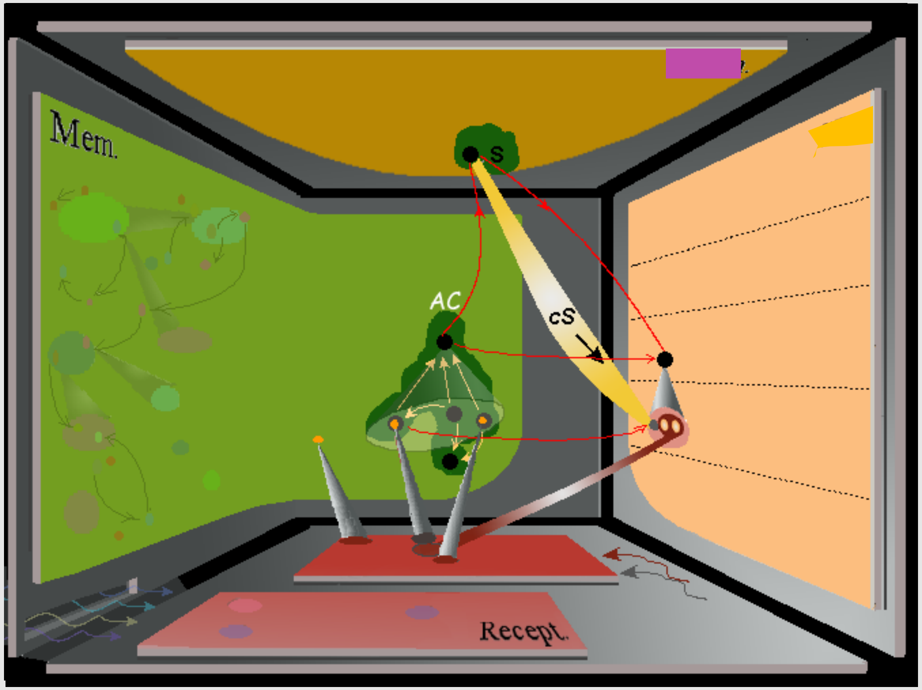
Internet sites where most papers can be downloaded

http://ehres.pagesperso-orange.fr

http://vbm-ehr.pagesperso-orange.fr

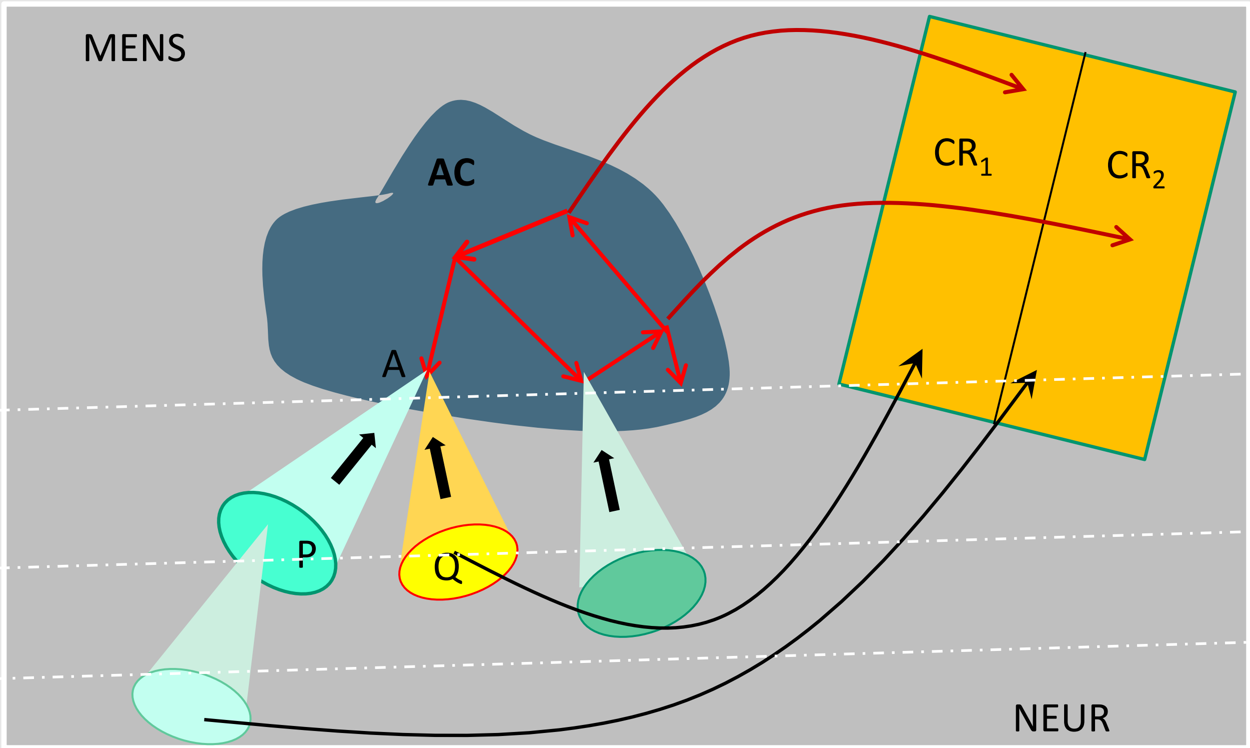
THANKS

THE ARCHETYPAL CORE AC



In **MENS** MP allows the development, over time, of *higher order memory components* integrating significant memories of various modalities, *with many ramifications and possibility of switches.* They constitute the *Archetpal Core* **AC**, a central subsystem of **Mem** in which they are connected by strong and fast links forming *archetypal loops* self-maintaining their activation for a long time. **AC** acts as a *dynamic flexible internal model*

GLOBAL LANDSCAPE

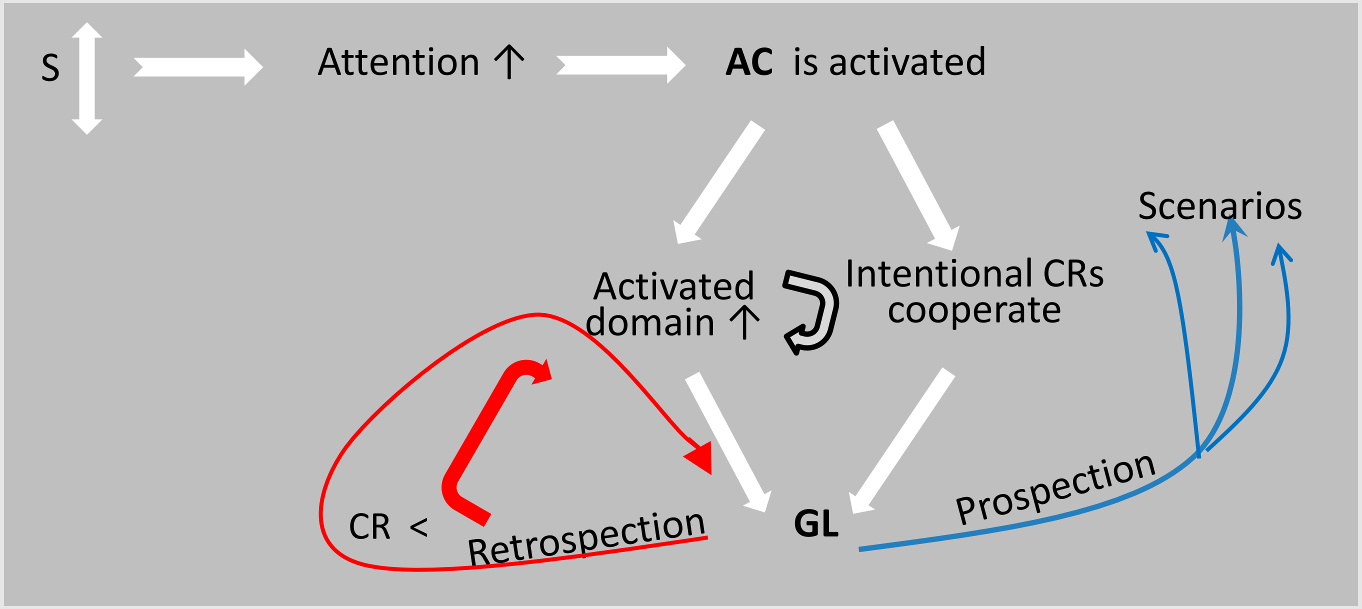


Intentional co-regulator = CR_i based on associative brain areas, linked to **AC**. Activation of part of **AC** diffuses through self-maintained archetypal loops.

It propagates to a decomposition P of some A, then, via a switch, to another decomposition Q of A and down a ramification.

Transmitted back to intentional CR_i, it allows the formation of a *global landscape* **GL** uniting and extending spatially and temporally their landscapes .

CONSCIOUS PROCESSES



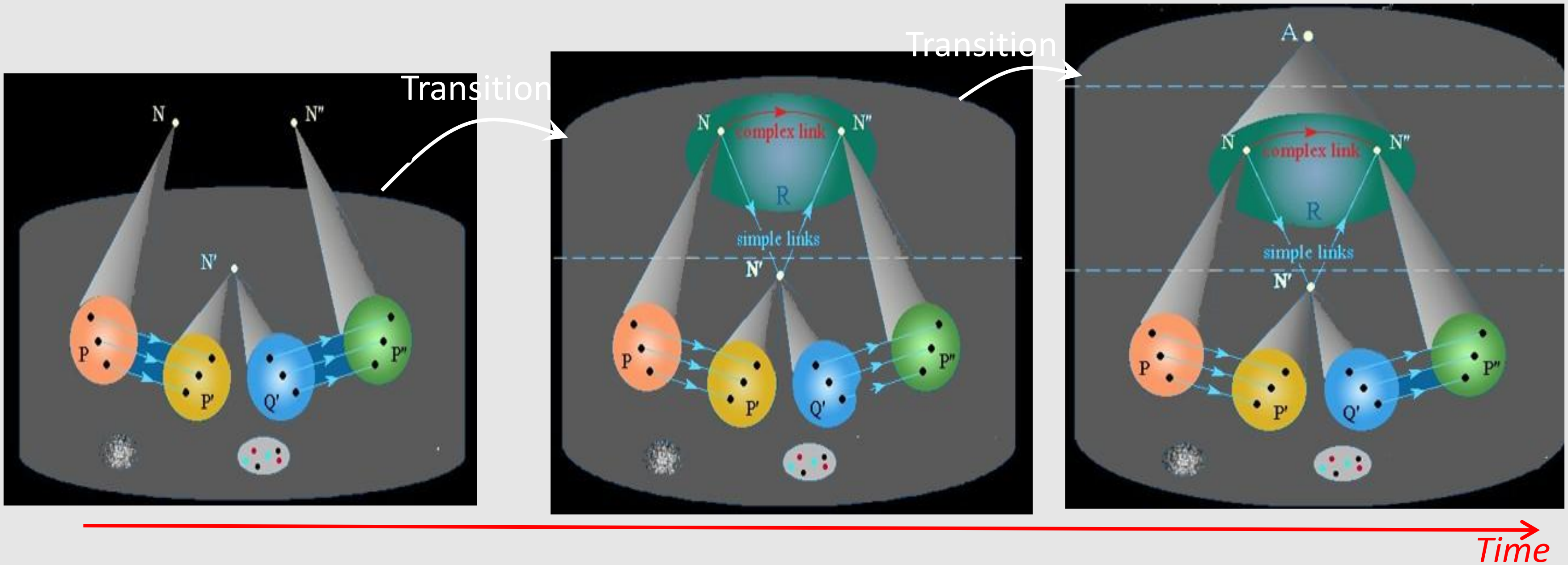
Unexpected or striking event S activates part of AC

===> formation of a long term global landscape **GL**, in which *conscio*us *processes* develop by integration of time via:

1. ***Retrospection:*** analysis of the situation and recall of near past for "making sense" of S (by abduction). Cp. "exploratory creativity" (Boden) ;

2. ***Prospection***: search of "scenarios" by iteratively constructing virtual landscapes ("mental spaces") in GL in which sequences of procedures are tried, by evaluation of the corresponding complexifications.

ANTICIPATION. CREATIVITY



They depend on overlapping retrospection/prospection processes in GLs.
'Simple' scenarios are obtained by complexification of a virtual landscape ('mental space’) in **GL** ===> "combinatory" creativity (Boden), "coherent blending" (Fauconnier & Turner).
More 'innovative' scenarios ("transformational" creativity of Boden) are obtained by iterated complexifications of virtual landscape, since:

THEOREM. ***A double complexification where complex links play a role cannot be reduced to a unique complexification, and it allows the emergence of components of increasing complexity order.***